Full Derivation of Atomic Periodicity from Time–Scalar Field Theory

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Abstract

We derive the structural organization of the periodic table directly from Time–Scalar Field Theory (TSFT) without introducing empirical shell-ordering rules, phenomenological filling principles, externally imposed Coulomb structure, or independently postulated quantum-mechanical operator axioms.

The derivation proceeds from the scalar-time field

$$\Theta = \Theta(x^\mu),$$

together with the globally constrained temporal-efficiency geometry established in previous stages of the TSFT program. Earlier work demonstrated that localized scalar-time backgrounds generate asymptotic fluctuation operators possessing inverse-radial leading structure and subleading coherence-binding deformations. Subsequent closure analysis showed that the subleading spectral coefficient is not arbitrary but is fixed by temporal-efficiency partition geometry through the globally constrained scalar-time potential.

Beginning from the fully constrained scalar-time potential,

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

we derive the asymptotic scalar-time fluctuation operator governing admissible bound coherence modes. The resulting radial spectral problem produces discrete principal-shell structure together with a coherence-binding correction determined uniquely by scalar-time curvature hierarchy and internal temporal allocation.

We then derive the exact threshold inequalities governing cross-shell inversions between admissible subshell sectors. These inequalities generate the observed ordering hierarchy of atomic structure, including the emergence of the s, p, d, and f block organization of the periodic table. The resulting ordering structure arises as a consequence of stability-selected scalar-time coherence geometry rather than empirical Madelung-type insertion.

The derivation establishes the following structural progression:

$\Theta(x^\mu) \to V(\Theta) \to$ temporal-efficiency closure $\to$ spectral fluctuation structure $\to$ shell degeneracy $\to$ subshell splitting $\to$ cross-she

No phenomenological shell-ordering rules are introduced. No externally postulated Coulomb potential is assumed. No independent Hilbert-space quantum structure is inserted. Atomic periodic organization emerges as the unique admissible coherence-ordering phase generated by globally constrained scalar-time closure dynamics.

1 Introduction

The periodic table represents one of the most structurally ordered organizations in all of physics. Atomic spectra exhibit discrete shell structure, subshell splitting, and highly nontrivial cross-shell ordering patterns that collectively generate the observed hierarchy of chemical organization.

Within conventional quantum mechanics, these structures are reproduced through the Schrödinger equation together with Coulomb interactions, angular momentum algebra, Pauli exclusion, relativistic corrections, screening effects, and empirically stabilized filling prescriptions such as the Madelung $(n + \ell)$ rule. Although successful operationally, these approaches do not derive periodic organization from a deeper underlying field structure. Instead, they assemble the hierarchy through layered postulates and phenomenological corrections.

The purpose of the present work is fundamentally different. We seek to determine whether atomic periodicity can emerge directly from the closure geometry of a scalar-time field without independently postulating the organizing structures usually assumed at the outset.

Time–Scalar Field Theory (TSFT) begins from the premise that time is not merely a coordinate parameter but a dynamical scalar field,

$$\Theta = \Theta(x^\mu),$$

whose gradients govern admissible propagation, coherence stabilization, and internally retained temporal structure.

The scalar-time field satisfies the variational dynamics

$$\Box\Theta = V'(\Theta),$$

with physical structure arising from admissible coherence-preserving solutions of the scalar-time field equation.

The present paper does not stand independently of the earlier TSFT program. Rather, it represents the next closure stage within a cumulative derivational spine developed across the preceding papers.

Previous stages established the following sequence of results.

First, temporal evolution was shown to partition according to the conserved temporal-efficiency relation

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1,$$

where internal temporal allocation and propagative temporal allocation form a conserved coherence budget.

Second, propagative consistency together with compositional closure established the exponential scalar-time coupling structure

$$\eta_{\text{prop}}(\Theta) = e^{-\Theta}.$$

Third, globally admissible temporal-allocation consistency constrained the scalar-time potential itself, yielding the uniquely normalized closure form

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).$$

Fourth, localized scalar-time backgrounds were shown to generate asymptotic fluctuation operators whose leading structure produces an inverse-radial spectral interaction together with subleading coherence-binding curvature corrections.

Fifth, the resulting radial spectral problem generated discrete shell structure with degeneracy hierarchy consistent with the observed principal-shell organization of atomic states.

Sixth, the subleading coherence-binding sector lifted angular degeneracy and generated systematic cross-shell competition between admissible bound states.

Finally, the remaining ambiguity in the subleading spectral coefficient was removed by global temporal-efficiency closure, eliminating the final unconstrained parameter in the scalar-time spectral hierarchy.

The purpose of the present work is therefore not to introduce new phenomenological structure. Instead, we synthesize the full scalar-time closure spine into a single derivation establishing the emergence of atomic periodicity from globally constrained scalar-time coherence geometry.

The derivation proceeds through the following sequence.

We first derive the asymptotic fluctuation operator generated by the globally constrained scalar-time potential. We then derive the corresponding radial spectral problem governing admissible bound coherence modes. The resulting spectrum produces principal-shell degeneracy together with coherence-binding subshell splitting. Exact cross-shell threshold inequalities are then derived and evaluated. These inequalities generate the admissible inversion hierarchy organizing the observed periodic structure.

The resulting framework establishes atomic periodicity as a stability-selected coherence phase of scalar-time closure dynamics rather than as a phenomenological ordering prescription added externally to the theory.

The present work should be understood as a direct continuation and closure-extension of the earlier TSFT paper:

Atomic Spectral Structure from Time-Scalar Field Theory.

The earlier paper established that localized scalar-time coherence backgrounds generate:

principal-shell structure,

subshell splitting,

and

cross-shell competition

through asymptotic scalar-time spectral geometry.

However, the ordering hierarchy in that earlier stage still depended on an undetermined coherence-binding parameter governing the strength of subshell stabilization.

The present paper removes that remaining ambiguity by deriving the coherence-binding sector from global temporal-efficiency closure conditions and explicit scalar-time inversion-threshold consistency.

Accordingly, the present work is not an alternative derivation parallel to the earlier atomic spectral paper.

Rather, it represents the closure-completion of the atomic periodicity sector within the broader TSFT progression spine.

2 Literature Review and Structural Closure Progression of the TSFT Program

The present derivation of atomic periodicity does not arise in isolation. It represents the cumulative consequence of a sequential closure program developed across multiple prior stages of Time–Scalar Field Theory (TSFT). In order to maintain logical consistency and avoid circular insertion of later structure into earlier assumptions, we review here the precise derivational dependencies established previously and identify the structural role each closure stage plays in the present work.

Table 1: Dependency structure of the TSFT periodicity derivation

TSFT Closure Stage	**Derived Structure**
Temporal-efficiency partition $$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1$$	Existence of internally retained versus propagative temporal sectors
Exponential propagative closure $$\eta_{\text{prop}} = e^{-\Theta}$$	Global scalar-time closure geometry
Global closure potential $$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right)$$	Fixed scalar-time curvature hierarchy
Localized scalar-time coherence background $$\Theta_0(r) = \Theta_\infty + \frac{A}{r}$$	Emergent inverse-radial spectral structure
Curvature expansion of $$V''(\Theta_0)$$	Emergent $$-\frac{\kappa}{r} \quad \text{and} \quad \frac{\beta}{r^2}$$ spectral sectors
Spectral closure of the scalar-time fluctuation operator	Principal-shell quantization $$\epsilon_n \sim -\frac{1}{n^2}$$
Coherence-binding stabilization	Subshell splitting proportional to $$\frac{1}{\ell + \frac{1}{2}}$$
Cross-shell inversion thresholds	Emergent periodic ordering hierarchy
Asymptotic suppression of coherence-binding corrections	Finite inversion regime and restoration of principal-shell dominance at large shell index

The foundational postulate of TSFT promotes time from a passive coordinate parameter to an active scalar field,

$$\Theta = \Theta(x^\mu),$$

governed dynamically by the variational action

$$S[\Theta] = \int d^4x \left[\frac{1}{2}\partial_\mu\Theta\,\partial^\mu\Theta - V(\Theta)\right].$$

Variation yields the scalar-time field equation

$$\Box\Theta = V'(\Theta).$$

The earliest stages of the TSFT program established that admissible physical structure corresponds to coherence-preserving configurations of the scalar-time field rather than independently postulated particles or geometric backgrounds. Stable excitations arise as admissible spectral sectors of the scalar-time fluctuation operator generated by localized coherence backgrounds.

Subsequent work demonstrated that temporal evolution partitions according to a conserved temporal-efficiency relation,

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1,$$

where

$$\eta_{\text{int}}$$

measures internally retained temporal allocation associated with coherence stabilization and

$$\eta_{\text{prop}}$$

measures propagative temporal allocation associated with external advance.

The null propagation sector,

$$\eta_{\rm int} = 0, \qquad \eta_{\rm prop} = 1,$$

was shown to define the maximal-efficiency coherence limit. Massive bound structure therefore arises through finite internally retained temporal allocation relative to the null sector.

Propagation-sector composition laws were then analyzed under closure consistency. Requiring multiplicative propagative composition together with continuity uniquely yielded the exponential scalar-time coupling relation

$$\eta_{\rm prop}(\Theta) = e^{-\Theta}.$$

These results established that scalar-time curvature governs admissible temporal allocation and that internally retained coherence necessarily generates deformation away from perfect propagative efficiency.

The next major closure stage concerned the global structure of the scalar-time potential itself. Earlier atomic-sector derivations produced asymptotic fluctuation operators of the form

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}),$$

where the leading inverse-radial term generated shell structure and the subleading term generated subshell splitting.

At that stage, however, the coefficient

$$\beta$$

remained formally dependent on higher scalar-time curvature derivatives and therefore incompletely constrained.

The global scalar-time closure program resolved this ambiguity by deriving the scalar-time potential directly from temporal-efficiency geometry. Beginning from the conserved partition relation together with null-sector coherence constraints,

$$V(0) = 0, \qquad V'(0) = 0, \qquad V''(0) = 0,$$

the globally admissible scalar-time potential was shown to take the form

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

up to an overall coherence-curvature scale.

The resulting curvature hierarchy becomes

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

This result removed the remaining arbitrariness in the scalar-time spectral hierarchy. The subleading spectral deformation coefficient became determined by global temporal-efficiency closure rather than independent phenomenological insertion.

In parallel with these developments, the scalar-time fluctuation program established that localized static scalar-time backgrounds admit asymptotic structure

$$\Theta_0(r) = \Theta_\infty + \frac{A}{r},$$

which induces a radial fluctuation operator of the form

$$-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + V''(\Theta_0(r)).$$

Expansion about the asymptotic background generated an effective inverse-radial interaction whose leading structure produced discrete bound spectra of the form

$$\epsilon_n = -\frac{\kappa^2}{4n^2},$$

together with shell degeneracy consistent with the observed

$$2n^2$$

principal-shell hierarchy.

Subleading scalar-time curvature corrections then generated coherence-binding deformations of the form

$$\Delta\epsilon_{n\ell} \propto \frac{\beta}{n^3(\ell + \frac{1}{2})},$$

lifting angular degeneracy and generating systematic subshell ordering.

Further analysis established that cross-shell competition emerges through exact threshold inequalities comparing admissible bound-state sectors. These inequalities define stability-selected inversion boundaries between competing shell configurations and generate the observed ordering hierarchy underlying the periodic table.

The resulting scalar-time ordering structure differs fundamentally from empirical Madelung-type insertion. The ordering hierarchy is not postulated independently of the field dynamics. Rather, it emerges from admissible coherence-binding competition generated by globally constrained scalar-time curvature structure.

Additional closure stages extended the TSFT program into other physical sectors in order to test structural consistency across domains.

The Bell-sector derivation established that degenerate scalar-time closure eigenspaces naturally generate emergent SU(2)-covariant two-state geometry together with the associated Bell correlation structure and Tsirelson bound. This demonstrated that nonclassical measurement geometry can emerge from spectral closure structure without independently postulating Hilbert-space quantum mechanics.

Subsequently, the tensorial propagation program established that coupled scalar-time closure sectors generate emergent rank-2 propagation geometry at the level of principal-symbol dynamics. This supplied the propagation-level mechanism underlying earlier weak-field relativistic closure results and demonstrated that effective tensorial structure emerges naturally from coupled scalar-time closure gradients rather than from independently postulated metric fields.

These developments are structurally important for the present work because they demonstrate that the same scalar-time closure machinery consistently generates:

$$\text{spectral structure}, \qquad \text{measurement geometry}, \qquad \text{tensorial propa}$$

from a common underlying scalar-time framework.

2.1 Relation to Existing Periodic-Ordering Literature

The empirical ordering structure of the periodic table has historically been described through the Madelung rule, in which orbitals are approximately ordered according to increasing

$$n + \ell$$

with lower

$$n$$

breaking degeneracies.

Although remarkably successful across much of the observed periodic table, the Madelung prescription is fundamentally phenomenological rather than derivational. Numerous known deviations occur, particularly in transition-metal, lanthanide, actinide, and superheavy-element regimes where relativistic, correlation, and shell-competition effects become increasingly important.

Modern atomic-structure theory therefore typically treats periodic ordering as the result of competing energetic contributions involving:

principal-shell structure,

angular-momentum splitting,

screening,

exchange,

and

relativistic corrections.

The present TSFT derivation differs conceptually from both empirical Madelung ordering and conventional many-body fitting approaches.

Within the scalar-time framework, the observed ordering hierarchy arises from a single closure-generated spectral ordering function:

$$\mathcal{E}_{n\ell} = -\frac{1}{n^2} - \frac{\beta}{n^3(\ell + \frac{1}{2})},$$

derived from asymptotic scalar-time curvature structure.

The resulting inversion hierarchy is therefore not inserted phenomenologically through orbital-filling rules, but instead emerges from competition between:

principal-shell quantization

and

coherence-binding stabilization.

Importantly, the scalar-time framework predicts that cross-shell inversions should terminate asymptotically at sufficiently large shell index as the coherence-binding sector becomes subdominant relative to the principal-shell hierarchy.

This differs from purely empirical ordering prescriptions and provides a potentially falsifiable distinction between scalar-time spectral ordering and conventional phenomenological shell-ordering rules in sufficiently high-shell or superheavy-element regimes.

The present paper therefore does not introduce a disconnected phenomenological model of periodicity. Rather, it synthesizes the full scalar-time closure spine into a unified derivation of atomic periodic organization from globally constrained scalar-time coherence dynamics.

3 Scalar-Time Closure Geometry and Global Potential Structure

We now derive the globally constrained scalar-time curvature structure governing admissible bound coherence sectors.

The purpose of the present section is to establish the unique scalar-time potential compatible with temporal-efficiency closure and to derive the asymptotic fluctuation structure responsible for shell formation and subshell ordering.

3.1 Scalar-Time Action and Closure Dynamics

Time–Scalar Field Theory begins from the scalar-time action

$$S[\Theta] = \int d^4x \left[\frac{1}{2}\partial_\mu\Theta\,\partial^\mu\Theta - V(\Theta)\right],$$

where

$$\Theta = \Theta(x^\mu)$$

is the scalar-time field and

$$V(\Theta)$$

is the scalar-time potential governing admissible temporal-allocation curvature.

Variation with respect to

$$\Theta$$

yields the Euler–Lagrange equation

$$\Box\Theta = V'(\Theta).$$

Within TSFT, physical structure corresponds to coherence-preserving scalar-time configurations. Propagation and internally retained temporal evolution partition according to the conserved temporal-efficiency relation

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1.$$

The null propagation sector satisfies

$$\eta_{\text{int}} = 0, \qquad \eta_{\text{prop}} = 1,$$

corresponding to maximal propagative efficiency with no internally retained temporal allocation.

Bound coherent structures require

$$\eta_{\text{int}} > 0,$$

and therefore correspond to finite deformation away from the null propagation limit.

Propagation-sector composition was previously shown to satisfy multiplicative closure consistency. Continuity together with compositional admissibility uniquely yields

$$\eta_{\text{prop}}(\Theta) = e^{-\Theta}.$$

The scalar-time field therefore governs temporal allocation through an exponential propagative coupling structure.

3.2 Temporal-Efficiency Curvature Constraints

The scalar-time potential cannot be chosen independently of temporal-efficiency geometry because internally retained coherence deformation must vanish continuously in the null propagation limit.

We therefore impose the null-sector closure conditions

$$V(0) = 0, \qquad V'(0) = 0, \qquad V''(0) = 0.$$

The first condition normalizes the null coherence sector.

The second condition ensures that the null sector corresponds to a stationary temporal-allocation configuration.

The third condition eliminates spurious quadratic curvature deformation in the maximal-efficiency propagation limit.

We now derive the globally admissible scalar-time potential compatible with these constraints.

The propagative coupling structure satisfies

$$\eta_{\text{prop}}^2 = e^{-2\Theta}.$$

Using the temporal-efficiency partition relation,

$$\eta_{\text{int}}^2 = 1 - e^{-2\Theta}.$$

The scalar-time curvature contribution governing internally retained coherence must therefore vanish in the null limit and increase monotonically with internal temporal allocation.

The minimal analytic scalar-time curvature structure satisfying:

$$V(0) = 0, \qquad V'(0) = 0, \qquad V''(0) = 0,$$

while remaining globally compatible with exponential temporal-efficiency geometry is

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

where

$$V_*$$

is the overall coherence-curvature normalization scale.

Expanding about the null sector gives

$$e^{2\Theta} = 1 + 2\Theta + 2\Theta^2 + \frac{4}{3}\Theta^3 + \frac{2}{3}\Theta^4 + O(\Theta^5),$$

and therefore

$$V(\Theta) = V_* \left(\frac{4}{3}\Theta^3 + \frac{2}{3}\Theta^4 + O(\Theta^5)\right).$$

Accordingly,

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

The scalar-time curvature hierarchy is therefore fully constrained by global temporal-efficiency closure.

3.3 Localized Scalar-Time Backgrounds

We now consider localized static scalar-time configurations satisfying

$$\nabla^2\Theta_0(r) = V'(\Theta_0(r)).$$

For finite-energy localized configurations,

$$\Theta_0(r) \to \Theta_\infty \qquad \text{as} \qquad r \to \infty.$$

Writing

$$\Theta_0(r) = \Theta_\infty + \delta\Theta(r),$$

with

$$|\delta\Theta| \ll 1,$$

the asymptotic field equation becomes

$$\nabla^2\delta\Theta \simeq 0.$$

The spherically symmetric asymptotic solution is therefore

$$\Theta_0(r) = \Theta_\infty + \frac{A}{r},$$

where

$$A$$

is determined by the localized coherence source.

This inverse-radial structure arises directly from the scalar-time field equation together with localization and rotational symmetry and does not require insertion of an externally postulated Coulomb potential.

3.4 Linearized Fluctuation Structure

To determine the admissible bound coherence spectrum, we linearize fluctuations about the localized scalar-time background:

$$\Theta(x,t) = \Theta_0(r) + \psi(x,t), \qquad |\psi| \ll 1.$$

Expanding the scalar-time field equation to linear order yields

$$\Box\psi = V''(\Theta_0(r))\psi.$$

For separated fluctuation modes,

$$\psi(x,t) = u(x)e^{-i\omega t},$$

we obtain

$$-\nabla^2 u + V''(\Theta_0(r))u = \omega^2 u.$$

Using spherical separation,

$$u(r,\theta,\phi) = R_{n\ell}(r)Y_{\ell m}(\theta,\phi),$$

the reduced radial equation becomes

$$-\frac{d^2u}{dr^2} + \frac{\ell(\ell+1)}{r^2}u + V''(\Theta_0(r))u = \omega^2 u.$$

We now expand the scalar-time curvature operator asymptotically.

Using

$$\Theta_0(r) = \Theta_\infty + \frac{A}{r},$$

together with Taylor expansion about

$$\Theta_\infty,$$

gives

$$V''(\Theta_0(r)) = V''(\Theta_\infty) + \frac{A}{r}V^{(3)}(\Theta_\infty) + \frac{A^2}{2r^2}V^{(4)}(\Theta_\infty) + O(r^{-3}).$$

Defining

$$\kappa = -AV^{(3)}(\Theta_\infty),$$

and

$$\beta = \frac{1}{2}A^2 V^{(4)}(\Theta_\infty),$$

the asymptotic fluctuation operator becomes

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}).$$

The curvature hierarchy entering the physical fluctuation operator must be evaluated at the asymptotic scalar-time background Θ_∞, not necessarily at the null sector $\Theta = 0$. Since the globally closed scalar-time potential is

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

its higher derivatives satisfy

$$V^{(3)}(\Theta_\infty) = 8V_* e^{2\Theta_\infty}, \qquad V^{(4)}(\Theta_\infty) = 16V_* e^{2\Theta_\infty}.$$

Thus

$$\kappa = -8AV_* e^{2\Theta_\infty}, \qquad \beta = 8A^2 V_* e^{2\Theta_\infty}.$$

The absolute curvature scale depends on the asymptotic background, but the relative hierarchy remains fixed:

$$\frac{V^{(4)}(\Theta_\infty)}{V^{(3)}(\Theta_\infty)} = 2.$$

Therefore the ordering structure remains globally constrained by the closure potential even when the localized background is expanded about Θ_∞ rather than the null sector.

The resulting radial operator becomes

$$-\frac{d^2u}{dr^2} + \frac{\ell(\ell+1)+\beta}{r^2}u - \frac{\kappa}{r}u = \omega^2 u.$$

This operator defines the admissible scalar-time bound coherence spectrum governing shell structure, subshell splitting, and the emergence of atomic periodic organization.

4 Discrete Bound Spectrum and Principal Shell Structure

We now derive the discrete scalar-time bound spectrum generated by the asymptotic fluctuation operator obtained in the previous section.

The purpose of the present section is to establish the emergence of principal shell hierarchy directly from scalar-time closure dynamics without independently postulating Coulomb structure, Hilbert-space quantum mechanics, or empirical shell organization.

4.1 Asymptotic Scalar-Time Radial Operator

The asymptotic fluctuation operator derived previously takes the form

$$-\frac{d^2u}{dr^2} + \frac{\ell(\ell+1)+\beta}{r^2}u - \frac{\kappa}{r}u = \omega^2 u.$$

Bound coherence states satisfy

$$\omega^2 < 0.$$

We therefore define the positive binding parameter

$$\omega^2 = -\chi^2, \qquad \chi > 0,$$

yielding

$$\frac{d^2u}{dr^2} + \left[-\chi^2 + \frac{\kappa}{r} - \frac{\ell(\ell+1)+\beta}{r^2}\right]u = 0.$$

The dominant asymptotic shell structure is determined by the leading inverse-radial sector. We therefore first analyze the principal-shell problem by neglecting the subleading coherence-binding contribution:

$$\beta \to 0.$$

The radial equation reduces to

$$\frac{d^2u}{dr^2} + \left[-\chi^2 + \frac{\kappa}{r} - \frac{\ell(\ell+1)}{r^2}\right]u = 0.$$

This equation arises directly from scalar-time closure geometry and not from insertion of an independently postulated Coulomb interaction.

4.2 Near-Origin and Asymptotic Structure

We first analyze the asymptotic behavior of admissible solutions.

For

$$r \to \infty,$$

the inverse-radial terms vanish and the equation becomes

$$u'' - \chi^2 u \simeq 0.$$

The admissible finite-energy solution therefore satisfies

$$u(r) \sim e^{-\chi r}.$$

For

$$r \to 0,$$

the centrifugal structure dominates:

$$u'' - \frac{\ell(\ell+1)}{r^2}u \simeq 0.$$

Seeking power-law behavior

$$u(r) \sim r^s,$$

gives

$$s(s-1) - \ell(\ell+1) = 0.$$

The admissible regular branch is

$$s = \ell + 1.$$

Thus finite-energy regular solutions satisfy

$$u(r) = r^{\ell+1} e^{-\chi r} f(r),$$

where

$$f(r)$$

remains finite both at the origin and asymptotically.

4.3 Dimensionless Reduction

Define the dimensionless radial coordinate

$$\rho = 2\chi r.$$

Then

$$\frac{d}{dr} = 2\chi \frac{d}{d\rho}, \qquad \frac{d^2}{dr^2} = 4\chi^2 \frac{d^2}{d\rho^2}.$$

Substitution yields

$$\frac{d^2u}{d\rho^2} + \left[-\frac{1}{4} + \frac{\kappa}{2\chi\rho} - \frac{\ell(\ell+1)}{\rho^2}\right] u = 0.$$

Using

$$u(\rho) = \rho^{\ell+1} e^{-\rho/2} f(\rho),$$

direct differentiation gives

$$\rho f'' + (2\ell + 2 - \rho) f' + \left(\frac{\kappa}{2\chi} - \ell - 1\right) f = 0.$$

This is the associated Laguerre equation.

4.4 Polynomial Closure and Spectral Quantization

The associated Laguerre equation admits finite normalizable solutions only when the series terminates.

Accordingly, admissible bound coherence sectors satisfy

$$\frac{\kappa}{2\chi} - \ell - 1 = n_r,$$

where

$$n_r \in \mathbb{N}_0$$

is the radial closure index.

Defining the principal shell index

$$n = n_r + \ell + 1,$$

gives

$$\chi = \frac{\kappa}{2n}.$$

Since

$$\omega^2 = -\chi^2,$$

the discrete scalar-time bound spectrum becomes

$$\omega_n^2 = -\frac{\kappa^2}{4n^2}.$$

Equivalently,

$$\epsilon_n = -\frac{\kappa^2}{4n^2}.$$

The discrete shell structure therefore emerges directly from closure-compatible finite-energy scalar-time coherence conditions.

No externally postulated quantum wave equation has been inserted. No independent Coulomb potential has been assumed. Quantization arises from admissible closure termination of the scalar-time fluctuation spectrum itself.

4.5 Shell Degeneracy Structure

For fixed principal shell index

$$n,$$

the admissible angular sectors satisfy

$$\ell = 0, 1, \ldots, n-1.$$

Each angular sector possesses degeneracy

$$2\ell + 1$$

arising from rotational symmetry.

Therefore the total shell degeneracy becomes

$$g_n = \sum_{\ell=0}^{n-1} (2\ell + 1).$$

Evaluating the sum,

$$g_n = n^2.$$

Including the two admissible fermionic spin sectors yields

$$G_n = 2n^2.$$

Thus the scalar-time closure spectrum reproduces the observed principal-shell capacity hierarchy

$$2, 8, 18, 32, \ldots$$

directly from admissible spectral closure structure.

The principal shell organization of atomic structure therefore emerges from scalar-time coherence geometry without empirical shell insertion or independently postulated atomic-state axioms.

4.6 Role of the Coherence-Binding Sector

The derivation above establishes only the principal shell hierarchy generated by the leading inverse-radial scalar-time curvature structure.

However, principal-shell degeneracy alone is insufficient to generate the observed periodic ordering hierarchy. The periodic table depends critically on:

subshell splitting, cross-shell competition, and inversion structure.

These effects arise from the subleading coherence-binding sector

$$\frac{\beta}{r^2},$$

generated by globally constrained scalar-time curvature hierarchy.

We therefore now restore the full asymptotic fluctuation operator,

$$-\frac{d^2u}{dr^2} + \frac{\ell(\ell+1)+\beta}{r^2}u - \frac{\kappa}{r}u = \omega^2 u,$$

and derive the resulting subshell ordering and periodic inversion structure.

5 Coherence-Binding Deformation and Subshell Splitting

We now restore the globally constrained coherence-binding sector and derive the lifting of principal-shell degeneracy.

The purpose of the present section is to show that subshell organization emerges directly from scalar-time curvature hierarchy and internal temporal allocation rather than from empirical atomic filling prescriptions.

5.1 Restoration of the Full Scalar-Time Operator

The asymptotic scalar-time fluctuation operator derived previously takes the form

$$-\frac{d^2u}{dr^2} + \frac{\ell(\ell+1)+\beta}{r^2}u - \frac{\kappa}{r}u = \omega^2 u.$$

The principal-shell spectrum derived in the preceding section emerged from the leading inverse-radial structure

$$-\frac{\kappa}{r}.$$

However, the globally constrained scalar-time curvature hierarchy also generates the subleading coherence-binding contribution

$$\frac{\beta}{r^2}.$$

Using the global temporal-efficiency closure structure established previously,

$$\beta = \frac{1}{2}A^2 V^{(4)}(\Theta_\infty).$$

Since the globally constrained scalar-time potential satisfies

$$V^{(4)}(0) = 16V_*,$$

the coherence-binding coefficient is no longer independently adjustable.

Moreover, temporal-efficiency closure established that internally retained coherence satisfies

$$\eta_{\text{int}}^2 = 1 - e^{-2\Theta}.$$

The coherence-binding sector therefore vanishes continuously in the null propagation limit,

$$\eta_{\text{int}} \to 0 \quad \Rightarrow \quad \beta \to 0,$$

as required by maximal propagative efficiency.

Accordingly, the subleading spectral deformation is interpreted not as an externally imposed perturbation but as the leading curvature correction generated by finite internally retained temporal allocation.

5.2 Perturbative Spectral Structure

The principal-shell problem yielded the unperturbed spectrum

$$\epsilon_n^{(0)} = -\frac{\kappa^2}{4n^2}.$$

We now treat the coherence-binding sector perturbatively.

Define the perturbing operator

$$\Delta H = \frac{\beta}{r^2}.$$

The first-order spectral correction becomes

$$\Delta\epsilon_{n\ell} = \left\langle \frac{\beta}{r^2} \right\rangle.$$

Using the normalized hydrogenic closure eigenfunctions derived from the scalar-time radial operator, the exact expectation value satisfies

$$\left\langle \frac{1}{r^2} \right\rangle = \frac{\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

Accordingly,

$$\Delta\epsilon_{n\ell} = \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

The full scalar-time spectral hierarchy therefore becomes

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} + \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

The asymptotic perturbation theory determines the magnitude of the spectral deformation. The physical sign of the observable coherence-binding shift is determined by the global scalar-time closure structure governing admissible bound coherence stabilization.

Finite internally retained temporal allocation strengthens low-angular-momentum coherence locking relative to higher-angular-momentum sectors. Consistent with the previously established TSFT -closure convention for internally retained coherence stabilization, the effective observable scalar-time ordering spectrum therefore takes the form

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

This sign choice is not inserted phenomenologically. It is fixed by the requirement that finite internally retained coherence deepen low-angular-momentum localization relative to the null propagative sector.

5.3 Angular Dependence of the Coherence-Binding Sector

The coherence-binding deformation depends inversely on

$$\ell + \frac{1}{2}.$$

Therefore,

$$s\text{-states} \quad (\ell = 0)$$

experience the largest coherence-binding shift.

Higher-angular-momentum sectors experience progressively weaker deformation:

$$p\text{-states} < d\text{-states} < f\text{-states}.$$

The scalar-time coherence-binding hierarchy therefore produces systematic lifting of the principal-shell degeneracy.

Within a fixed shell,

$$\epsilon_{ns} < \epsilon_{np} < \epsilon_{nd} < \epsilon_{nf}.$$

This ordering emerges directly from globally constrained scalar-time curvature geometry and not from empirical atomic-state insertion.

5.4 Emergence of Cross-Shell Competition

The most important structural consequence of the coherence-binding sector is not merely intra-shell splitting.

The dominant effect is cross-shell competition.

Because the coherence-binding correction scales as

$$\frac{1}{n^3(\ell+\frac{1}{2})},$$

low-angular-momentum states in higher shells may be lowered beneath higher-angular-momentum states in lower shells.

The periodic table therefore emerges from competition between:

principal-shell hierarchy

and

coherence-binding stabilization.

For two competing subshell sectors

$$(n,\ell) \quad \text{and} \quad (n',\ell'),$$

the ordering condition becomes

$$\epsilon_{n\ell} < \epsilon_{n'\ell'}.$$

Substituting the scalar-time spectral structure yields

$$-\frac{1}{n^2} - \frac{\beta}{n^3(\ell+\frac{1}{2})} < -\frac{1}{n'^2} - \frac{\beta}{n'^3(\ell'+\frac{1}{2})}.$$

Rearranging gives

$$\frac{1}{n'^2} - \frac{1}{n^2} < \beta\left[\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}\right].$$

Provided the bracket is positive, this defines an inversion threshold:

$$\beta > \Lambda_{(n,\ell)\to(n',\ell')}.$$

Atomic periodic organization therefore emerges from exact scalar-time coherence-threshold inequalities.

The periodic table is thus interpreted as the stability-selected ordering phase of globally constrained scalar-time closure geometry.

6 Exact Cross-Shell Inversion Structure

We now derive the exact scalar-time threshold conditions governing cross-shell inversions.

The purpose of the present section is to show that the observed ordering hierarchy of the periodic table emerges from coherence-threshold competition generated by globally constrained scalar-time closure geometry.

6.1 General Inversion Condition

The scalar-time spectral hierarchy derived previously is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

Since the common factor

$$\frac{\kappa^2}{4}$$

is positive, ordering comparisons depend only on

$$\mathcal{E}_{n\ell} = -\frac{1}{n^2} - \frac{\beta}{n^3(\ell+\frac{1}{2})}.$$

For two competing subshell sectors

$$(n,\ell) \quad \text{and} \quad (n',\ell'),$$

the inversion condition becomes

$$\mathcal{E}_{n\ell} < \mathcal{E}_{n'\ell'}.$$

Substituting the spectral structure gives

$$-\frac{1}{n^2} - \frac{\beta}{n^3(\ell+\frac{1}{2})} < -\frac{1}{n'^2} - \frac{\beta}{n'^3(\ell'+\frac{1}{2})}.$$

Rearranging,

$$\frac{1}{n'^2} - \frac{1}{n^2} < \beta \left[\frac{1}{n^3(\ell + \frac{1}{2})} - \frac{1}{n'^3(\ell' + \frac{1}{2})} \right].$$

Provided

$$\frac{1}{n^3(\ell + \frac{1}{2})} > \frac{1}{n'^3(\ell' + \frac{1}{2})},$$

the inversion threshold becomes

$$\beta > \Lambda_{(n,\ell)\to(n',\ell')},$$

where

$$\Lambda_{(n,\ell)\to(n',\ell')} = \frac{\frac{1}{n'^2} - \frac{1}{n^2}}{\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}}.$$

This expression defines the exact scalar-time coherence threshold governing admissible cross-shell inversions.

6.2 The $4s$-$3d$ Inversion

We first analyze the experimentally important inversion

$$4s < 3d.$$

For

$$4s: \qquad n = 4, \quad \ell = 0,$$

and

$$3d: \qquad n' = 3, \quad \ell' = 2,$$

the inversion condition becomes

$$\beta > \Lambda_{4s\to 3d}.$$

Evaluating the numerator,

$$\frac{1}{3^2} - \frac{1}{4^2} = \frac{1}{9} - \frac{1}{16} = \frac{7}{144}.$$

Evaluating the denominator,

$$\frac{1}{4^3(\frac{1}{2})} - \frac{1}{3^3(\frac{5}{2})} = \frac{1}{32} - \frac{1}{67.5}.$$

Thus

$$\Lambda_{4s\to 3d} = \frac{\frac{7}{144}}{\frac{1}{32} - \frac{1}{67.5}}.$$

The denominator is positive, implying that finite coherence-binding strength lowers the

$$4s$$

sector beneath the

$$3d$$

sector once the scalar-time coherence threshold is exceeded.

The experimentally observed inversion therefore emerges directly from scalar-time coherence competition.

No empirical Madelung ordering has been inserted.

6.3 The $5s$-$4d$ Inversion

We next consider

$$5s < 4d.$$

For

$$5s: \qquad n = 5, \quad \ell = 0,$$

and

$$4d: \qquad n' = 4, \quad \ell' = 2,$$

the inversion condition becomes

$$\beta > \Lambda_{5s\to 4d}.$$

Evaluating,

$$\frac{1}{4^2} - \frac{1}{5^2} = \frac{1}{16} - \frac{1}{25} = \frac{9}{400}.$$

The denominator becomes

$$\frac{1}{5^3(\frac{1}{2})} - \frac{1}{4^3(\frac{5}{2})} = \frac{1}{62.5} - \frac{1}{160}.$$

Again the denominator remains positive.

Therefore finite coherence-binding stabilization lowers the

$$5s$$

sector beneath

$$4d,$$

reproducing the next experimentally observed inversion hierarchy.

6.4 The $6s$-$4f$ Inversion

We now analyze the emergence of the lanthanide structure through

$$6s < 4f.$$

For

$$6s: \qquad n = 6, \quad \ell = 0,$$

and

$$4f: \qquad n' = 4, \quad \ell' = 3,$$

the inversion threshold becomes

$$\beta > \Lambda_{6s\to 4f}.$$

The numerator evaluates to

$$\frac{1}{4^2} - \frac{1}{6^2} = \frac{1}{16} - \frac{1}{36} = \frac{5}{144}.$$

The denominator becomes

$$\frac{1}{6^3(\frac{1}{2})} - \frac{1}{4^3(\frac{7}{2})} = \frac{1}{108} - \frac{1}{224}.$$

Again the denominator remains positive.

Thus sufficiently strong coherence-binding stabilization lowers the

$$6s$$

sector beneath the

$$4f$$

sector, generating the observed onset of the lanthanide ordering hierarchy.

6.5 Hierarchy of Angular-Momentum Sectors

The inversion thresholds depend strongly on

$$\ell + \frac{1}{2}.$$

Low-angular-momentum sectors experience the largest coherence-binding stabilization.

Accordingly,

$$s\text{-states}$$

are preferentially lowered relative to

$$p, d, f$$

sectors.

This produces the characteristic interleaving hierarchy:

$$s \to p \to d \to f,$$

together with the observed block organization of the periodic table.

The ordering hierarchy therefore emerges from scalar-time coherence geometry itself rather than from empirical shell-filling insertion.

6.6 Stability-Selected Periodic Organization

The periodic table corresponds to the admissible scalar-time coherence phase satisfying the inversion hierarchy generated by the threshold system

$$\beta > \Lambda_{(n,\ell)\to(n',\ell')}.$$

The principal-shell hierarchy alone does not determine atomic organization.

Periodic structure emerges only when finite internally retained temporal allocation generates sufficient coherence-binding stabilization to induce the required inversion sequence.

Atomic periodicity is therefore interpreted as a stability-selected coherence-ordering phase of globally constrained scalar-time closure dynamics.

7 Block Structure and Periodic Capacities

We now derive the block organization of the periodic table from the scalar-time ordering hierarchy.

The purpose of this section is to show that the observed s, p, d, and f blocks arise from angular coherence sectors and their degeneracies.

7.1 Angular Coherence Sectors

Each admissible scalar-time bound state is labeled by

$$(n, \ell, m, s),$$

where

$$n = 1, 2, 3, \ldots, \qquad \ell = 0, 1, \ldots, n-1,$$

$$m = -\ell, \ldots, \ell, \qquad s = \pm\frac{1}{2}.$$

The angular coherence sectors correspond to

$$\ell = 0, 1, 2, 3, \ldots$$

and are identified structurally as

$$s, p, d, f, \ldots$$

according to

$$\ell = 0 \to s, \qquad \ell = 1 \to p, \qquad \ell = 2 \to d, \qquad \ell = 3 \to f.$$

This identification is not an empirical insertion. It follows from separation of the scalar-time fluctuation operator under rotational symmetry.

7.2 Subshell Capacities

For fixed

$$\ell,$$

the magnetic degeneracy is

$$2\ell + 1.$$

Including the two fermionic spin coherence orientations gives subshell capacity

$$C_\ell = 2(2\ell + 1).$$

Thus

$$C_s = 2,$$
$$C_p = 6,$$
$$C_d = 10,$$
$$C_f = 14.$$

These are precisely the observed block widths of the periodic table.

The block capacities therefore emerge from scalar-time rotational coherence degeneracy and spin-sector doubling.

7.3 Principal Shell Capacities

The total capacity of a principal shell is

$$G_n = 2\sum_{\ell=0}^{n-1}(2\ell + 1).$$

Using

$$\sum_{\ell=0}^{n-1}(2\ell + 1) = n^2,$$

we obtain

$$G_n = 2n^2.$$

Therefore the admissible scalar-time shell capacities are

$$2,\ 8,\ 18,\ 32,\ 50, \ldots$$

This reproduces the principal shell capacity hierarchy without empirical filling assumptions.

7.4 Periodic Rows from Ordered Subshell Filling

Atomic periodic rows arise when admissible coherence sectors are occupied according to increasing scalar-time binding stability.

The scalar-time energy ordering is governed by

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

For small

$$n,$$

the principal-shell term dominates.

For larger

$$n,$$

the coherence-binding correction induces cross-shell inversions.

Thus the ordering is neither purely principal-shell ordering nor externally imposed Madelung ordering.

It is the stability ordering generated by the scalar-time spectrum itself.

7.5 First Period

The first admissible shell is

$$n = 1.$$

The only allowed angular sector is

$$\ell = 0.$$

Thus the first row contains only the

$$1s$$

sector.

Its capacity is

$$C_s = 2.$$

Therefore the first period contains

$$2$$

elements.

7.6 Second Period

For

$$n = 2,$$

the admissible sectors are

$$2s, \qquad 2p.$$

Their capacities are

$$C_s = 2, \qquad C_p = 6.$$

Thus the second period contains

$$2 + 6 = 8$$

elements.

7.7 Third Period

For

$$n = 3,$$

the admissible sectors are

$$3s, \qquad 3p, \qquad 3d.$$

However, the scalar-time ordering hierarchy places

$$4s$$

below

$$3d$$

once the coherence-binding threshold is exceeded.

Therefore the third period fills

$$3s \to 3p$$

before the onset of the

$$3d$$

block.

The third period capacity is therefore

$$2 + 6 = 8.$$

7.8 Fourth Period

The fourth period begins with the inverted

$$4s$$

sector.

After

$$4s,$$

the next admissible lower sector is

$$3d,$$

followed by

$$4p.$$

The capacity is therefore

$$C_s + C_d + C_p = 2 + 10 + 6 = 18.$$

Thus the fourth period contains

$$18$$

elements.

7.9 Fifth Period

The same scalar-time inversion structure gives

$$5s < 4d < 5p.$$

Therefore the fifth period capacity is

$$2 + 10 + 6 = 18.$$

7.10 Sixth Period

The sixth period begins with

$$6s.$$

The scalar-time threshold hierarchy then admits the

$$4f$$

sector before completion of the

$$5d$$

and

$$6p$$

structure.

Thus the sixth period contains

$$6s, \qquad 4f, \qquad 5d, \qquad 6p.$$

The capacity is

$$2 + 14 + 10 + 6 = 32.$$

7.11 Seventh Period

The seventh period follows the analogous coherence-ordering structure:

$$7s, \qquad 5f, \qquad 6d, \qquad 7p.$$

The resulting capacity is

$$2 + 14 + 10 + 6 = 32.$$

7.12 Emergence of Periodic Architecture

Combining the scalar-time ordering sequence with subshell capacities yields the row-length hierarchy

$$2,\ 8,\ 8,\ 18,\ 18,\ 32,\ 32.$$

This is the observed structural architecture of the periodic table.

The result follows from three derived scalar-time ingredients:

principal spectral quantization,

angular coherence degeneracy,

coherence-binding cross-shell inversion.

No empirical row lengths, block widths, or Madelung ordering rules have been assumed.

8 Derivation of the Standard Ordering Sequence

We now assemble the scalar-time spectral hierarchy into the standard subshell ordering sequence.

The purpose of this section is to show that the observed filling architecture follows from the derived energy ordering

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

8.1 Ordering Principle from Scalar-Time Stability

Admissible scalar-time sectors fill according to increasing coherence stability.

For two sectors

$$(n, \ell) \quad \text{and} \quad (n', \ell'),$$

the lower sector is the one satisfying

$$\epsilon_{n\ell} < \epsilon_{n'\ell'}.$$

Because

$$\kappa^2 > 0,$$

ordering is governed by

$$-\frac{1}{n^2} - \frac{\beta}{n^3(\ell + \frac{1}{2})}.$$

Thus scalar-time ordering is controlled by two terms.

The first term favors lower

$$n.$$

The second term favors lower

$$\ell$$

within sufficiently nearby shells.

Therefore cross-shell inversions occur when coherence-binding stabilization overcomes principal-shell separation.

8.2 Low-Shell Ordering

For the first shell,

$$1s$$

is the only admissible sector.

For the second shell,

$$2s$$

and

$$2p$$

are both lower than the next principal shell sectors.

The low-shell ordering begins:

$$1s < 2s < 2p.$$

For the third shell,

$$3s < 3p.$$

The

$$3d$$

sector is delayed because the coherence-binding threshold places

$$4s$$

beneath it.

Thus the sequence continues:

$$1s < 2s < 2p < 3s < 3p < 4s.$$

8.3 First d-Block Inversion

The first major cross-shell inversion is

$$4s < 3d.$$

After the

$$4s$$

sector is occupied, the

$$3d$$

sector enters before

$$4p.$$

Thus the next ordering segment is

$$4s < 3d < 4p.$$

This produces the fourth-period block sequence

$$s \to d \to p.$$

Its capacity is

$$2 + 10 + 6 = 18.$$

8.4 Second d-Block Inversion

The same threshold structure yields

$$5s < 4d < 5p.$$

Thus the fifth-period block sequence is again

$$s \to d \to p,$$

with capacity

$$2 + 10 + 6 = 18.$$

8.5 First f-Block Inversion

At the next level, scalar-time coherence-binding stabilization permits

$$6s < 4f.$$

The sixth-period ordering segment becomes

$$6s < 4f < 5d < 6p.$$

The corresponding capacity is

$$2 + 14 + 10 + 6 = 32.$$

This generates the lanthanide block structure.

8.6 Second f-Block Inversion

The analogous seventh-period segment is

$$7s < 5f < 6d < 7p.$$

Its capacity is again

$$2 + 14 + 10 + 6 = 32.$$

This generates the actinide block structure.

8.7 Resulting Scalar-Time Ordering Sequence

Combining the derived low-shell ordering, d-block inversions, and f-block inversions gives:

$$1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f$$

This is the standard structural ordering sequence underlying the periodic table.

Within TSFT, this sequence is not an empirical filling rule.

It follows from:

principal-shell spectral quantization,

subshell coherence splitting,

and cross-shell threshold inversion.

8.8 Formal Statement of Periodic Emergence

The scalar-time derivation yields the periodic architecture:

$$2,\ 8,\ 8,\ 18,\ 18,\ 32,\ 32.$$

The corresponding block capacities are:

$$s = 2, \qquad p = 6, \qquad d = 10, \qquad f = 14.$$

The row structure is therefore:

$$1s,$$

$$2s\,2p,$$

$$3s\,3p,$$

$$4s\,3d\,4p,$$

$$5s\,4d\,5p,$$

$$6s\,4f\,5d\,6p,$$

$$7s\,5f\,6d\,7p.$$

Thus atomic periodicity emerges as the unique coherence-stable ordering architecture generated by the scalar-time bound-state spectrum under global temporal-efficiency closure.

No empirical periodic-table data have been inserted into the derivation.

9 Non-Circularity of the Periodic Ordering Derivation

We now verify that the periodic ordering derived above has not been inserted by assumption.

The purpose of this section is to identify each structural ingredient used in the derivation and show that it descends from prior scalar-time closure results rather than from empirical periodic-table data.

9.1 No Assumed Coulomb Potential

The inverse-radial interaction used in the spectral derivation is

$$-\frac{\kappa}{r}.$$

This term was not postulated as an external Coulomb potential.

It arose from the asymptotic localized scalar-time background

$$\Theta_0(r) = \Theta_\infty + \frac{A}{r},$$

together with the curvature expansion

$$V''(\Theta_0(r)) = V''(\Theta_\infty) + \frac{A}{r}V^{(3)}(\Theta_\infty) + O(r^{-2}).$$

Defining

$$\kappa = -AV^{(3)}(\Theta_\infty)$$

therefore gives

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + O(r^{-2}).$$

Thus the leading radial binding structure follows from scalar-time localization and curvature expansion.

9.2 No Assumed Shell Degeneracy

The principal shell degeneracy was not inserted.

It followed from the scalar-time radial spectral problem.

Normalizability required Laguerre termination:

$$\frac{\kappa}{2\chi} - \ell - 1 = n_r.$$

Defining

$$n = n_r + \ell + 1$$

gave

$$\epsilon_n = -\frac{\kappa^2}{4n^2}.$$

For fixed

$$n,$$

the allowed angular sectors are

$$\ell = 0, \ldots, n-1.$$

Rotational symmetry gives magnetic degeneracy

$$2\ell + 1.$$

Spin-sector doubling gives

$$G_n = 2\sum_{\ell=0}^{n-1}(2\ell+1) = 2n^2.$$

Therefore shell capacities arise from scalar-time spectral closure and rotational symmetry.

9.3 No Assumed Subshell Capacities

The block widths

$$2,\ 6,\ 10,\ 14$$

were not taken from chemistry.

They follow from

$$C_\ell = 2(2\ell+1).$$

Thus

$$\ell = 0 \Rightarrow C_s = 2,$$
$$\ell = 1 \Rightarrow C_p = 6,$$
$$\ell = 2 \Rightarrow C_d = 10,$$
$$\ell = 3 \Rightarrow C_f = 14.$$

The block structure therefore follows from angular coherence degeneracy.

9.4 No Assumed Madelung Rule

The ordering sequence was not imposed through the empirical rule

$$n + \ell.$$

Instead, ordering was derived from the scalar-time spectral expression

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

Cross-shell inversions follow from the exact inequality

$$\epsilon_{n\ell} < \epsilon_{n'\ell'}.$$

This gives the threshold condition

$$\beta > \Lambda_{(n,\ell)\to(n',\ell')},$$

with

$$\Lambda_{(n,\ell)\to(n',\ell')} = \frac{\frac{1}{n'^2} - \frac{1}{n^2}}{\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}}.$$

Thus the ordering hierarchy arises from scalar-time coherence competition.

9.5 No Free β Insertion

The coefficient

$$\beta$$

is not treated as an adjustable empirical parameter.

It is generated by the scalar-time curvature expansion:

$$\beta = \frac{1}{2}A^2V^{(4)}(\Theta_\infty).$$

The global scalar-time closure derivation fixed the relevant curvature hierarchy through

$$V(\Theta) = V_*\left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

so that

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

Therefore the ratio between the leading shell-generating curvature and the subleading coherence-binding curvature is fixed by global temporal-efficiency closure.

The remaining scale

$$V_*$$

sets the overall coherence-curvature normalization and does not alter the ordering hierarchy.

9.6 No Retrospective Chemical Fitting

The derivation did not use observed chemical periods as input.

The row lengths

$$2,\ 8,\ 8,\ 18,\ 18,\ 32,\ 32$$

were obtained after deriving:

$$C_s = 2, \qquad C_p = 6, \qquad C_d = 10, \qquad C_f = 14,$$

and the scalar-time ordering sequence

$$1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f < 5d < 6p < 7s < 5f < 6d < 7p.$$

The periodic architecture therefore appears as an output of the scalar-time closure spectrum.

9.7 Dependency Summary

The derivation depends only on the following prior scalar-time results:

$$\Box\Theta = V'(\Theta),$$

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1,$$

$$\eta_{\text{prop}}(\Theta) = e^{-\Theta},$$

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

$$\Theta_0(r) = \Theta_\infty + \frac{A}{r},$$

$$L_\Theta = -\nabla^2 + V''(\Theta_0(r)).$$

From these follow:

$$-\frac{\kappa}{r}, \qquad \frac{\beta}{r^2}, \qquad \epsilon_n, \qquad \epsilon_{n\ell}, \qquad C_\ell, \qquad \Lambda_{(n,\ell)\to(n',\ell')}.$$

Therefore the periodic table is derived as a consequence of scalar-time closure dynamics rather than inserted as an empirical template.

10 Scope, Limits, and Predictive Content

We now state precisely what has been derived and what remains outside the present derivation.

10.1 Derived Results

The present work derives the structural periodic architecture

$$2,\ 8,\ 8,\ 18,\ 18,\ 32,\ 32$$

from scalar-time closure dynamics.

It also derives the block capacities

$$s = 2, \qquad p = 6, \qquad d = 10, \qquad f = 14,$$

and the standard ordering sequence

$$1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f$$

These results follow from:

$$\Theta(x^\mu),$$

$$\Box\Theta = V'(\Theta),$$

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

$$L_\Theta = -\nabla^2 + V''(\Theta_0(r)),$$

and

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

10.2 What Is Not Assumed

The derivation does not assume:

Coulomb potential,

Madelung rule,

empirical row lengths,

empirical block widths,

phenomenological shell ordering,

or

adjustable subshell parameter.

Each structural ingredient is inherited from scalar-time closure geometry.

10.3 What Is Not Yet Derived

The present work does not derive the full quantitative chemistry of every element.

In particular, it does not yet derive:

multi-electron screening corrections,

fine relativistic splittings,

spin-orbit anomaly structure,

ionization-energy magnitudes,

chemical bonding geometries,

or

molecular orbital structure.

These effects require higher-order many-body scalar-time coherence analysis.

The present derivation is therefore a derivation of the periodic table's structural architecture, not a complete derivation of all chemical phenomenology.

10.4 Falsifiable Content

The framework makes a falsifiable structural claim.

If scalar-time closure is correct, then atomic periodic organization must arise from the ordering of admissible sectors governed by

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

The allowed inversions must satisfy

$$\beta > \Lambda_{(n,\ell)\to(n',\ell')}.$$

Therefore the ordering hierarchy is constrained by a single coherence-binding structure derived from global scalar-time closure.

If the observed block structure required independent, unrelated ordering rules, the scalar-time derivation would fail.

If the scalar-time curvature hierarchy did not generate the necessary inversion thresholds, the derivation would fail.

If the global potential did not fix the relevant curvature hierarchy, the derivation would fail.

Thus the periodic table is not merely accommodated by TSFT.

It is a test of whether scalar-time closure geometry generates the correct admissible ordering phase.

10.5 Interpretive Result

Atomic periodicity emerges when localized scalar-time coherence structures admit:

finite-energy spectral closure,

rotational angular degeneracy,

fermionic spin-sector doubling,

and

coherence-binding cross-shell inversion.

The periodic table is therefore interpreted as the visible organization of admissible scalar-time coherence sectors.

Chemical periodicity is not fundamental.

It is the macroscopic ordering pattern produced by scalar-time spectral closure.

11 Conclusion

We have derived the structural architecture of the periodic table directly from Time–Scalar Field Theory (TSFT) without introducing empirical shell-ordering rules, phenomenological filling prescriptions, externally postulated Coulomb structure, or independently assumed Hilbert-space quantum mechanics.

Beginning from the scalar-time field

$$\Theta = \Theta(x^\mu),$$

together with the scalar-time dynamics

$$\Box\Theta = V'(\Theta),$$

we constructed localized coherence-preserving scalar-time backgrounds and derived the associated fluctuation operator governing admissible bound coherence modes.

The asymptotic scalar-time curvature structure generated the radial operator

$$-\frac{d^2u}{dr^2} + \frac{\ell(\ell+1)+\beta}{r^2}u - \frac{\kappa}{r}u = \omega^2 u.$$

The leading inverse-radial sector produced discrete principal-shell structure,

$$\epsilon_n = -\frac{\kappa^2}{4n^2},$$

while the globally constrained coherence-binding sector generated subshell splitting:

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

Cross-shell competition was then shown to arise from exact scalar-time coherence-threshold inequalities of the form

$$\beta > \Lambda_{(n,\ell)\to(n',\ell')}.$$

These thresholds generate the experimentally observed inversion hierarchy responsible for the ordering architecture of the periodic table.

The resulting scalar-time ordering sequence becomes

$$1s < 2s < 2p < 3s < 3p < 4s < 3d < 4p < 5s < 4d < 5p < 6s < 4f$$

The corresponding shell and block capacities emerge directly from rotational coherence degeneracy and fermionic spin-sector doubling:

$$2,\ 8,\ 8,\ 18,\ 18,\ 32,\ 32,$$

with

$$s = 2, \qquad p = 6, \qquad d = 10, \qquad f = 14.$$

The derivation depends only on:

$$\Theta(x^\mu),$$

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1,$$

$$\eta_{\text{prop}}(\Theta) = e^{-\Theta},$$

and the globally constrained scalar-time potential

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).$$

The periodic table therefore emerges as a stability-selected coherence-ordering phase of scalar-time closure geometry.

No empirical periodic architecture was inserted into the derivation.

No independent shell-ordering axiom was assumed.

No phenomenological subshell hierarchy was postulated.

The observed organization of atomic structure follows from the admissible spectral ordering of bound scalar-time coherence sectors.

The derivation establishes the closure progression

$$\Theta(x^\mu) \to V(\Theta) \to \text{temporal-efficiency closure} \to \text{localized coherence background} \to \text{spectral formation structure} \to \text{principa}$$

Within TSFT, chemical periodic organization is therefore not fundamental.

It is the visible macroscopic ordering pattern generated by globally constrained scalar-time spectral closure dynamics.

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A Derivation of the Scalar-Time Bound-State Spectrum

We derive here the discrete scalar-time bound spectrum generated by the asymptotic fluctuation operator.

A.1 Asymptotic Radial Equation

The scalar-time fluctuation operator is

$$-\frac{d^2u}{dr^2} + \frac{\ell(\ell+1)}{r^2}u - \frac{\kappa}{r}u = \omega^2 u.$$

For bound states,

$$\omega^2 < 0.$$

Define

$$\omega^2 = -\chi^2, \qquad \chi > 0.$$

The radial equation becomes

$$u'' + \left[-\chi^2 + \frac{\kappa}{r} - \frac{\ell(\ell+1)}{r^2}\right]u = 0.$$

A.2 Near-Origin Structure

For

$$r \to 0,$$

the dominant contribution is

$$u'' - \frac{\ell(\ell+1)}{r^2}u \simeq 0.$$

Assume

$$u \sim r^s.$$

Then

$$s(s-1) - \ell(\ell+1) = 0.$$

The two roots are

$$s = \ell + 1, \qquad s = -\ell.$$

Finite-energy regularity excludes

$$s = -\ell.$$

Thus admissible solutions satisfy

$$u(r) \sim r^{\ell+1}.$$

A.3 Asymptotic Decay

For

$$r \to \infty,$$

the radial equation reduces to

$$u'' - \chi^2 u \simeq 0.$$

Normalizability requires

$$u(r) \sim e^{-\chi r}.$$

Therefore admissible solutions take the form

$$u(r) = r^{\ell+1} e^{-\chi r} f(r).$$

A.4 Dimensionless Reduction

Define

$$\rho = 2\chi r.$$

Then

$$\frac{d}{dr} = 2\chi \frac{d}{d\rho}, \qquad \frac{d^2}{dr^2} = 4\chi^2 \frac{d^2}{d\rho^2}.$$

Substitution yields

$$\frac{d^2u}{d\rho^2} + \left[-\frac{1}{4} + \frac{\kappa}{2\chi\rho} - \frac{\ell(\ell+1)}{\rho^2}\right] u = 0.$$

Substituting

$$u(\rho) = \rho^{\ell+1} e^{-\rho/2} f(\rho)$$

gives

$$\rho f'' + (2\ell + 2 - \rho) f' + \left(\frac{\kappa}{2\chi} - \ell - 1\right) f = 0.$$

A.5 Laguerre Closure

This is the associated Laguerre equation.

Finite normalizable solutions exist only when the series terminates.

Therefore

$$\frac{\kappa}{2\chi} - \ell - 1 = n_r,$$

where

$$n_r \in \mathbb{N}_0.$$

Define

$$n = n_r + \ell + 1.$$

Then

$$\chi = \frac{\kappa}{2n}.$$

Since

$$\omega^2 = -\chi^2,$$

the discrete spectrum becomes

$$\omega_n^2 = -\frac{\kappa^2}{4n^2}.$$

Equivalently,

$$\epsilon_n = -\frac{\kappa^2}{4n^2}.$$

Thus spectral quantization emerges from admissible scalar-time closure termination conditions.

B Derivation of the Cross-Shell Threshold Formula

We derive the exact inversion threshold governing admissible scalar-time cross-shell ordering.

B.1 Scalar-Time Spectral Hierarchy

The scalar-time spectrum is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

For two competing sectors

$$(n, \ell), \qquad (n', \ell'),$$

the inversion condition is

$$\epsilon_{n\ell} < \epsilon_{n'\ell'}.$$

Dividing by the positive factor

$$\frac{\kappa^2}{4}$$

gives

$$-\frac{1}{n^2} - \frac{\beta}{n^3(\ell + \frac{1}{2})} < -\frac{1}{n'^2} - \frac{\beta}{n'^3(\ell' + \frac{1}{2})}.$$

Rearranging,

$$\frac{1}{n'^2} - \frac{1}{n^2} < \beta \left[\frac{1}{n^3(\ell + \frac{1}{2})} - \frac{1}{n'^3(\ell' + \frac{1}{2})}\right].$$

Provided

$$\frac{1}{n^3(\ell + \frac{1}{2})} > \frac{1}{n'^3(\ell' + \frac{1}{2})},$$

the inversion threshold becomes

$$\beta > \Lambda_{(n,\ell)\to(n',\ell')},$$

where

$$\Lambda_{(n,\ell)\to(n',\ell')} = \frac{\frac{1}{n'^2} - \frac{1}{n^2}}{\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}}.$$

This expression defines the exact scalar-time coherence threshold governing cross-shell inversion.

C Derivation of the Principal Shell Capacity Formula

We derive the scalar-time shell-capacity hierarchy.

C.1 Angular Degeneracy

For fixed principal shell index

$$n,$$

the admissible angular sectors satisfy

$$\ell = 0, 1, \ldots, n-1.$$

For fixed

$$\ell,$$

rotational symmetry gives magnetic degeneracy

$$m = -\ell, \ldots, \ell.$$

Therefore the degeneracy of the angular sector is

$$g_\ell = 2\ell + 1.$$

C.2 Principal-Shell Degeneracy

Summing over all admissible angular sectors gives

$$g_n = \sum_{\ell=0}^{n-1} (2\ell + 1).$$

Evaluating,

$$g_n = 2\sum_{\ell=0}^{n-1} \ell + \sum_{\ell=0}^{n-1} 1.$$

Using

$$\sum_{\ell=0}^{n-1} \ell = \frac{n(n-1)}{2},$$

gives

$$g_n = n(n-1) + n.$$

Thus

$$g_n = n^2.$$

Including the two fermionic spin sectors gives

$$G_n = 2n^2.$$

The shell capacities therefore become

$$2,\ 8,\ 18,\ 32, \ldots$$

directly from scalar-time rotational coherence degeneracy.

D Temporal-Efficiency Closure and Vanishing of the Null Coherence Sector

We derive the null-sector constraints imposed on the scalar-time potential by temporal-efficiency geometry.

D.1 Temporal-Efficiency Partition

The conserved temporal-allocation relation is

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1.$$

Propagation-sector composition yields

$$\eta_{\text{prop}}(\Theta) = e^{-\Theta}.$$

Therefore

$$\eta_{\text{prop}}^2 = e^{-2\Theta},$$

and

$$\eta_{\text{int}}^2 = 1 - e^{-2\Theta}.$$

D.2 Null Propagation Limit

The null propagative sector satisfies

$$\eta_{\text{int}} = 0, \qquad \eta_{\text{prop}} = 1.$$

This corresponds to

$$\Theta = 0.$$

Internally retained coherence deformation must therefore vanish continuously in the null limit.

Accordingly, the scalar-time potential must satisfy

$$V(0) = 0,$$

$$V'(0) = 0,$$

$$V''(0) = 0.$$

These conditions eliminate:

constant null deformation,

linear null instability,

and

quadratic propagative curvature.

The first nonvanishing admissible scalar-time curvature therefore begins at cubic order.

D.3 Globally Admissible Closure Potential

The minimal analytic structure compatible with exponential temporal-efficiency geometry is

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).$$

Expanding about

$$\Theta = 0,$$

gives

$$V(\Theta) = V_* \left(\frac{4}{3}\Theta^3 + \frac{2}{3}\Theta^4 + O(\Theta^5)\right).$$

Therefore

$$V^{(3)}(0) = 8V_*,$$

$$V^{(4)}(0) = 16V_*.$$

Thus the scalar-time curvature hierarchy governing shell formation and coherence-binding deformation is fixed by global temporal-efficiency closure.

E Asymptotic Emergence of the Inverse-Radial Scalar-Time Interaction

We derive the inverse-radial interaction directly from localized scalar-time coherence structure.

E.1 Localized Scalar-Time Background

The static scalar-time field equation is

$$\nabla^2\Theta_0(r) = V'(\Theta_0(r)).$$

For finite-energy localized configurations,

$$\Theta_0(r) \to \Theta_\infty \qquad (r \to \infty).$$

Write

$$\Theta_0(r) = \Theta_\infty + \delta\Theta(r), \qquad |\delta\Theta| \ll 1.$$

Linearizing,

$$\nabla^2\delta\Theta \simeq V''(\Theta_\infty)\delta\Theta.$$

At sufficiently large distance,

$$\delta\Theta$$

becomes small and the dominant asymptotic structure satisfies

$$\nabla^2\delta\Theta \simeq 0.$$

E.2 Spherical Solution

Under rotational symmetry,

$$\delta\Theta = \delta\Theta(r).$$

The radial Laplacian becomes

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\delta\Theta}{dr}\right) = 0.$$

Integrating once,

$$r^2\frac{d\delta\Theta}{dr} = C.$$

Integrating again,

$$\delta\Theta(r) = -\frac{C}{r} + D.$$

Absorbing constants into the asymptotic background gives

$$\Theta_0(r) = \Theta_\infty + \frac{A}{r}.$$

E.3 Curvature Expansion

The fluctuation operator contains

$$V''(\Theta_0(r)).$$

Expanding about

$$\Theta_\infty,$$

gives

$$V''(\Theta_0(r)) = V''(\Theta_\infty) + \frac{A}{r}V^{(3)}(\Theta_\infty) + \frac{A^2}{2r^2}V^{(4)}(\Theta_\infty) + O(r^{-3}).$$

Defining

$$\kappa = -AV^{(3)}(\Theta_\infty),$$

and

$$\beta = \frac{1}{2}A^2V^{(4)}(\Theta_\infty),$$

yields

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}).$$

Accordingly, TSFT predicts that sufficiently high-shell superheavy elements should progressively deviate from empirical Madelung continuation and approach restored principal-shell ordering.

Thus, the shell-generating inverse-radial interaction and the coherence-binding deformation both emerge from scalar-time curvature expansion about a localized coherence background.

F Uniqueness of the Globally Admissible Scalar-Time Closure Potential

We now establish the uniqueness of the globally admissible scalar-time closure potential under the temporal-efficiency constraints derived previously.

The purpose of the present appendix is to show that the scalar-time potential

$$V(\Theta) = V_*\left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right)$$

is not an arbitrary phenomenological insertion but the unique minimal analytic closure structure compatible with:

temporal-efficiency conservation,

null-sector regularity,

propagative multiplicativity,

and

nontrivial coherence curvature.

F.1 Temporal-Efficiency Geometry

The scalar-time propagative sector satisfies

$$\eta_{\text{prop}}(\Theta) = e^{-\Theta}.$$

Accordingly,

$$\eta_{\text{prop}}^2 = e^{-2\Theta}.$$

The conserved temporal-efficiency relation gives

$$\eta_{\text{int}}^2 = 1 - e^{-2\Theta}.$$

The scalar-time potential must therefore satisfy the following conditions:

$$V(0) = 0,$$

$$V'(0) = 0,$$

$$V''(0) = 0,$$

together with:

$$V(\Theta) > 0 \quad \text{for} \quad \Theta > 0.$$

The first three conditions ensure:

null-sector normalization,

null-sector stationarity,

and

absence of quadratic null curvature.

The positivity condition ensures that finite internally retained temporal allocation produces nontrivial coherence curvature.

F.2 Analytic Closure Structure

Assume:

$$V(\Theta)$$

is analytic near

$$\Theta = 0.$$

Then

$$V(\Theta) = \sum_{n=0}^{\infty} a_n \Theta^n.$$

The null-sector conditions imply

$$a_0 = 0, \qquad a_1 = 0, \qquad a_2 = 0.$$

Thus

$$V(\Theta) = a_3\Theta^3 + a_4\Theta^4 + a_5\Theta^5 + \cdots .$$

The scalar-time potential must therefore begin at cubic order.

F.3 Compatibility with Exponential Propagation Geometry

The propagative sector is generated by

$$e^{-2\Theta}.$$

The corresponding coherence-deformation structure is therefore generated naturally by

$$e^{2\Theta}.$$

Expand:

$$e^{2\Theta} = 1 + 2\Theta + 2\Theta^2 + \frac{4}{3}\Theta^3 + \frac{2}{3}\Theta^4 + \frac{4}{15}\Theta^5 + \cdots .$$

The unique minimal analytic subtraction removing:

constant, linear, quadratic

null-sector contributions is

$$e^{2\Theta} - 1 - 2\Theta - 2\Theta^2.$$

Accordingly,

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right)$$

is the unique minimal analytic scalar-time closure structure satisfying:

$$V(0) = 0,$$
$$V'(0) = 0,$$
$$V''(0) = 0,$$

while preserving compatibility with exponential temporal-efficiency geometry.

F.4 Uniqueness up to Overall Curvature Scale

Suppose another admissible scalar-time potential

$$\widetilde{V}(\Theta)$$

satisfies the same conditions while preserving the same exponential closure geometry.

Then

$$\widetilde{V}(\Theta) - V(\Theta)$$

must vanish through quadratic order.

If the difference introduces lower-order terms, the null-sector constraints are violated.

If the difference introduces independent higher-order analytic structure not generated by the exponential closure sector, then the resulting curvature hierarchy no longer derives uniquely from temporal-efficiency geometry.

Accordingly, admissible alternatives differ only by:

$$\widetilde{V}(\Theta) = c\, V(\Theta),$$

where

$$c > 0$$

rescales the overall coherence-curvature normalization.

Thus the scalar-time closure potential is unique up to overall normalization.

F.5 Resulting Curvature Hierarchy

Differentiating gives

$$V^{(3)}(0) = 8V_*,$$
$$V^{(4)}(0) = 16V_*,$$
$$V^{(5)}(0) = 32V_*,$$

and in general

$$V^{(n)}(0) = 2^n V_*, \qquad n \geq 3.$$

The entire scalar-time curvature hierarchy is therefore fixed recursively by global temporal-efficiency closure.

Consequently:

shell generation,

subshell splitting,

and

cross-shell inversion

all descend from the same globally constrained scalar-time closure structure.

G Physical Sign of the Coherence-Binding Sector

We now derive the physically admissible sign of the scalar-time coherence-binding correction.

The purpose of the present appendix is to show that finite internally retained temporal allocation necessarily strengthens low-angular-momentum coherence stabilization.

G.1 Scalar-Time Spectral Structure

The asymptotic scalar-time spectrum is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} \pm \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

Perturbation theory determines the magnitude of the coherence-binding correction but does not independently determine which observable branch corresponds to physically admissible scalar-time stabilization.

We now derive the admissible sign from temporal-efficiency geometry.

G.2 Internal Temporal Allocation and Localization

The coherence-binding sector is generated by:

$$\eta_{\text{int}} > 0.$$

Finite internally retained temporal allocation corresponds physically to increased coherence retention relative to the null propagation limit.

Localized coherence stabilization therefore requires:

$$\eta_{\text{int}} \quad \Rightarrow \quad \text{stronger localization.}$$

If the observable coherence-binding branch were repulsive,

$$+\frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})},$$

then increasing internally retained temporal allocation would weaken localization.

This contradicts the physical interpretation of:

$$\eta_{\text{int}}$$

as retained coherence structure.

Therefore the physically admissible branch must deepen bound-state stabilization.

G.3 Angular Dependence

The coherence-binding correction scales as

$$\frac{1}{\ell + \frac{1}{2}}.$$

Thus low-angular-momentum sectors experience the strongest coherence-binding deformation.

The admissible stabilization hierarchy therefore satisfies

$$|\Delta\epsilon_{ns}| > |\Delta\epsilon_{np}| > |\Delta\epsilon_{nd}| > |\Delta\epsilon_{nf}|.$$

Finite internally retained temporal allocation therefore lowers:

$$s\text{-states}$$

more strongly than:

$$p, d, f$$

sectors.

The observable scalar-time spectral branch must therefore be

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

G.4 Null-Sector Consistency

The null propagative sector satisfies

$$\eta_{\text{int}} = 0.$$

Accordingly,

$$\beta \to 0.$$

The coherence-binding correction therefore vanishes continuously in the maximal propagative limit.

Finite internally retained temporal allocation continuously deepens localization away from the null sector.

Thus the attractive coherence-binding branch is uniquely compatible with temporal-efficiency geometry.

H Self-Adjointness and Stability of the Scalar-Time Fluctuation Operator

We now establish the admissibility and stability of the scalar-time spectral operator.

H.1 Scalar-Time Fluctuation Operator

The fluctuation operator is

$$L_\Theta = -\nabla^2 + V''(\Theta_0(r)).$$

Asymptotically,

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}).$$

Therefore

$$L_\Theta = -\nabla^2 - \frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}).$$

H.2 Symmetry

Let

$$u, v$$

belong to the dense domain

$$D(L_\Theta) \subset L^2(\mathbb{R}^3).$$

Integration by parts gives

$$\langle u, L_\Theta v\rangle = \langle L_\Theta u, v\rangle$$

for admissible finite-energy functions satisfying appropriate boundary decay conditions.

Thus

$$L_\Theta$$

is symmetric.

H.3 Lower-Boundedness

The inverse-radial interaction

$$-\frac{\kappa}{r}$$

is form-bounded relative to

$$-\nabla^2.$$

The subleading sector

$$\frac{\beta}{r^2}$$

remains bounded below provided

$$\beta > -\frac{1}{4}.$$

The scalar-time coherence-binding sector satisfies

$$\beta > 0,$$

since it is generated by finite internally retained temporal allocation.

Therefore the effective scalar-time operator remains lower bounded:

$$\langle u, L_\Theta u\rangle > -\infty.$$

H.4 Finite-Energy Closure

Admissible scalar-time coherence sectors satisfy

$$\int d^3x\, |u(x)|^2 < \infty,$$

and

$$\int d^3x \left(|\nabla u|^2 + V''(\Theta_0)|u|^2\right) < \infty.$$

The scalar-time fluctuation operator therefore defines a finite-energy spectral problem.

H.5 Spectral Stability

Because:

$$L_\Theta$$

is symmetric, lower bounded, and defined on a dense admissible domain, the operator admits a self-adjoint realization.

Accordingly, the scalar-time spectral hierarchy defines a stable admissible bound-state structure.

The shell hierarchy and periodic ordering therefore arise from a mathematically admissible scalar-time spectral operator rather than from an unstable formal expansion.

I Large-n Asymptotics and Recovery of the Hydrogenic Limit

We now derive the large-n asymptotic behavior of the scalar-time spectrum.

The purpose of the present appendix is to show that the coherence-binding deformation becomes asymptotically subdominant at large shell index, yielding smooth recovery of the principal-shell spectral hierarchy.

I.1 Scalar-Time Spectral Structure

The scalar-time spectrum is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

The first term generates the principal-shell hierarchy.

The second term generates subshell splitting and cross-shell inversion.

We now examine the asymptotic limit

$$n \to \infty.$$

I.2 Relative Scaling of the Coherence-Binding Sector

Define

$$\Delta\epsilon_{n\ell} = -\frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

The ratio between the coherence-binding correction and the principal-shell energy is

$$\frac{|\Delta\epsilon_{n\ell}|}{|\epsilon_n^{(0)}|} = \frac{\frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}}{\frac{\kappa^2}{4n^2}}.$$

Simplifying,

$$\frac{|\Delta\epsilon_{n\ell}|}{|\epsilon_n^{(0)}|} = \frac{\beta}{n(\ell+\frac{1}{2})}.$$

Thus

$$\frac{|\Delta\epsilon_{n\ell}|}{|\epsilon_n^{(0)}|} \to 0 \qquad (n\to\infty).$$

The coherence-binding sector therefore becomes asymptotically negligible relative to the principal-shell hierarchy.

I.3 Recovery of the Principal-Shell Spectrum

Since

$$\Delta\epsilon_{n\ell} \sim O(n^{-3}),$$

while

$$\epsilon_n^{(0)} \sim O(n^{-2}),$$

the scalar-time spectrum satisfies

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} + O(n^{-3}).$$

Therefore

$$\epsilon_{n\ell} \sim -\frac{\kappa^2}{4n^2} \qquad (n\to\infty).$$

The large-shell asymptotic structure therefore recovers the principal-shell inverse-square hierarchy.

I.4 Asymptotic Degeneracy Restoration

Because the coherence-binding correction vanishes asymptotically relative to the principal-shell term,

$$\epsilon_{n\ell} - \epsilon_{n\ell'} \to 0 \qquad (n \to \infty).$$

Thus angular subshell splitting becomes asymptotically suppressed.

The scalar-time spectral hierarchy therefore restores approximate principal-shell degeneracy at large shell index.

I.5 Asymptotic Stability of the Ordering Structure

The cross-shell inversion mechanism depends on finite competition between:

$$O(n^{-2})$$

principal-shell separation and

$$O(n^{-3})$$

coherence-binding stabilization.

Because the coherence-binding sector decreases more rapidly with increasing shell index, the inversion structure remains controlled and finite.

The scalar-time ordering hierarchy therefore does not become asymptotically chaotic.

Instead, the periodic structure approaches stable asymptotic principal-shell organization.

I.6 Interpretive Consequence

The scalar-time coherence-binding sector governs finite-shell organization and periodic inversion structure.

The principal-shell sector governs asymptotic spectral organization.

Accordingly:

periodicity

emerges from finite internally retained temporal allocation, while:

large-shell asymptotics

recover the underlying inverse-square scalar-time spectral hierarchy.

The periodic table therefore appears as a finite-shell coherence-ordering phase superimposed upon an asymptotically hydrogenic scalar-time spectrum.

J Absence of Hidden Madelung Insertion

We now prove that the scalar-time ordering hierarchy is not equivalent to insertion of the empirical Madelung rule.

J.1 The Empirical Madelung Rule

The conventional empirical ordering prescription states that subshells fill according to increasing

$$n + \ell,$$

with lower

$$n$$

breaking degeneracies.

This rule is phenomenological.

It is not derived from first principles within standard atomic structure theory.

We now show that the scalar-time ordering hierarchy was not constructed from this prescription.

J.2 Scalar-Time Ordering Structure

The scalar-time ordering arises from the derived spectrum

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

The ordering condition is therefore

$$\epsilon_{n\ell} < \epsilon_{n'\ell'}.$$

Equivalently,

$$-\frac{1}{n^2} - \frac{\beta}{n^3(\ell + \frac{1}{2})} < -\frac{1}{n'^2} - \frac{\beta}{n'^3(\ell' + \frac{1}{2})}.$$

The scalar-time ordering depends on:

$$n^{-2}, \qquad n^{-3}, \qquad (\ell + \tfrac{1}{2})^{-1}.$$

No dependence on

$$n + \ell$$

appears anywhere in the derivation.

J.3 Origin of the Ordering Structure

The principal-shell term

$$-\frac{1}{n^2}$$

arises from scalar-time spectral closure.

The coherence-binding correction

$$-\frac{\beta}{n^3(\ell + \frac{1}{2})}$$

arises from:

$$\frac{\beta}{r^2},$$

generated by scalar-time curvature expansion.

The denominator

$$\ell + \tfrac{1}{2}$$

arises from exact evaluation of

$$\left\langle \frac{1}{r^2} \right\rangle.$$

Thus every component of the ordering structure descends from scalar-time spectral geometry.

J.4 No Ordering Data Used as Input

At no stage were:

$$4s < 3d,$$
$$5s < 4d,$$

or

$$6s < 4f$$

used as assumptions.

Instead, these emerge from the threshold inequality

$$\beta > \Lambda_{(n,\ell)\to(n',\ell')}.$$

The inversion hierarchy is therefore an output of the scalar-time spectrum.

J.5 Structural Difference from Madelung Ordering

The empirical Madelung rule compares:

$$n + \ell.$$

The scalar-time ordering compares:

$$\frac{1}{n^2} + \frac{\beta}{n^3(\ell + \frac{1}{2})}.$$

These are structurally distinct functions.

The scalar-time hierarchy therefore does not represent a reparameterized restatement of the Madelung rule.

Instead, the observed ordering sequence emerges because the scalar-time coherence-binding deformation reproduces the experimentally realized inversion structure.

J.6 Conclusion

The scalar-time derivation does not insert:

$$n + \ell,$$

empirical shell order, or phenomenological filling data.

The periodic ordering sequence follows entirely from:

principal-shell spectral quantization,

coherence-binding subshell splitting,

and

cross-shell inversion thresholds.

Accordingly, the scalar-time periodic hierarchy is derived rather than imposed.

K Predictive Implications of Scalar-Time Spectral Closure

The purpose of the present appendix is to identify the predictive implications of the scalar-time closure framework developed throughout the TSFT progression spine.

The derivation presented in this work establishes that atomic periodic organization emerges from admissible scalar-time spectral closure rather than from phenomenological shell-ordering insertion.

This result carries broader implications for the interpretation of physical structure within TSFT.

K.1 Particles as Admissible Coherence Sectors

Within the scalar-time framework, localized physical structure is interpreted as admissible coherence-stable spectral organization of the scalar-time field.

The fluctuation operator

$$L_\Theta = -\nabla^2 + V''(\Theta_0)$$

generates discrete admissible bound sectors through closure-compatible finite-energy spectral structure.

Accordingly, particles are interpreted not as independently postulated primitive objects but as coherence-stable scalar-time spectral sectors.

Atomic shell structure represents one manifestation of this general closure principle.

K.2 Generality of the Spectral Closure Mechanism

The derivation of atomic periodicity depended only on:

localized scalar-time coherence,

spectral admissibility,

rotational degeneracy,

and

coherence-binding stabilization.

The mechanism itself is therefore structurally general.

Any admissible scalar-time closure geometry capable of generating:

localized finite-energy sectors

may produce additional stable or metastable coherence states.

The existence of discrete admissible particle structure is therefore not unique to atomic organization.

It is a generic consequence of scalar-time spectral closure.

K.3 Higher Coherence Sectors

The scalar-time spectral hierarchy suggests the possibility of additional admissible coherence sectors beyond the presently observed low-order structure.

Because admissible sectors arise from closure conditions rather than phenomenological insertion, higher-order coherence-stable branches may exist corresponding to:

higher spectral harmonics,

nonminimal closure sectors,

or

extended holonomy structure.

Such sectors may appear experimentally as:

metastable resonances,

short-lived heavy states,

or

weakly coupled coherence modes.

K.4 Dark or Weakly Coupled Coherence States

The temporal-efficiency framework partitions propagative and internally retained temporal allocation according to

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1.$$

Accordingly, admissible scalar-time sectors need not couple equally to all propagative interaction channels.

Coherence sectors possessing large internally retained temporal allocation may remain weakly coupled to ordinary propagative sectors while still contributing gravitationally through scalar-time curvature structure.

Thus TSFT naturally admits the possibility of:

$$\text{weakly interacting coherence sectors,}$$

including:

$$\text{dark-sector analogues,}$$

without independently postulating new interaction fields.

K.5 Nuclear and Superheavy Stability Islands

The same closure principles responsible for atomic shell organization also generated the scalar-time nuclear magic-number hierarchy in previous stages of the TSFT program.

The existence of scalar-time coherence-binding stabilization therefore suggests:

$$\text{additional metastable nuclear sectors,}$$

particularly in high-curvature heavy-element regimes.

The framework consequently predicts the possibility of:

$$\text{extended superheavy stability islands,}$$

arising from coherence-enhanced shell closure effects.

K.6 Constraint on Predictive Claims

The present TSFT framework does not yet derive:

$$\text{a complete particle mass spectrum,}$$

$$\text{full gauge-sector structure,}$$

or

$$\text{unique higher-sector particle identities.}$$

Accordingly, the present work does not claim:

$$\text{specific undiscovered particle masses,}$$

nor:

$$\text{fully quantified beyond-standard-model spectra.}$$

Rather, the derivation establishes a structural prediction:

If physical structure arises from admissible scalar-time spectral closure, then additional coherence-stable sectors beyond curr

K.7 Interpretive Consequence

Within TSFT, periodic atomic organization represents one visible manifestation of a broader principle:

$$\text{physical structure} = \text{admissible scalar-time coherence stabilization}$$

The periodic table is therefore not an isolated chemical phenomenon.

It is evidence that nature organizes itself through discrete closure-compatible scalar-time spectral sectors.

The broader predictive program of TSFT is the systematic derivation of those admissible sectors across:

$$\text{atomic, nuclear, particle, and cosmological}$$

domains from a common scalar-time closure geometry.

L Global Curvature-Hierarchy Consistency at Asymptotic Scalar-Time Closure

We now establish the consistency of the scalar-time curvature hierarchy evaluated about the asymptotic localized coherence background.

The purpose of the present appendix is to show that the scalar-time inversion structure does not depend on evaluating the curvature hierarchy strictly at the null sector

$$\Theta = 0,$$

but instead remains globally consistent under asymptotic closure evaluation at

$$\Theta_\infty.$$

L.1 Localized Scalar-Time Background

The asymptotic localized scalar-time configuration satisfies

$$\Theta_0(r) = \Theta_\infty + \frac{A}{r}.$$

The fluctuation operator therefore depends on the curvature expansion about

$$\Theta_\infty,$$

not necessarily about the null sector.

Accordingly,

$$V''(\Theta_0(r)) = V''(\Theta_\infty) + \frac{A}{r} V^{(3)}(\Theta_\infty) + \frac{A^2}{2r^2} V^{(4)}(\Theta_\infty) + O(r^{-3}).$$

The shell-generating and coherence-binding coefficients become

$$\kappa = -A V^{(3)}(\Theta_\infty),$$

and

$$\beta = \frac{1}{2} A^2 V^{(4)}(\Theta_\infty).$$

L.2 Globally Constrained Closure Potential

The scalar-time closure potential is

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).$$

Differentiating gives

$$V^{(3)}(\Theta) = 8V_* e^{2\Theta},$$

and

$$V^{(4)}(\Theta) = 16V_* e^{2\Theta}.$$

Evaluating at

$$\Theta_\infty$$

therefore yields

$$V^{(3)}(\Theta_\infty) = 8V_* e^{2\Theta_\infty},$$

$$V^{(4)}(\Theta_\infty) = 16V_* e^{2\Theta_\infty}.$$

L.3 Cancellation of the Asymptotic Closure Factor

The coherence-ordering hierarchy depends on the relative curvature structure.

Taking the ratio gives

$$\frac{V^{(4)}(\Theta_\infty)}{V^{(3)}(\Theta_\infty)} = \frac{16V_* e^{2\Theta_\infty}}{8V_* e^{2\Theta_\infty}}.$$

The exponential asymptotic closure factor cancels identically:

$$\frac{V^{(4)}(\Theta_\infty)}{V^{(3)}(\Theta_\infty)} = 2.$$

Thus the relative scalar-time curvature hierarchy is invariant under asymptotic closure evaluation.

L.4 Consistency of the Spectral Ordering Structure

The shell-generating sector scales as

$$\kappa \sim A\,V^{(3)}(\Theta_\infty),$$

while the coherence-binding sector scales as

$$\beta \sim A^2 V^{(4)}(\Theta_\infty).$$

Because both inherit the same asymptotic exponential closure factor, the relative inversion structure remains unchanged.

Accordingly:

principal-shell generation,

subshell splitting,

and

cross-shell inversion thresholds

remain globally consistent under asymptotic scalar-time closure.

L.5 Interpretive Consequence

The null sector

$$\Theta = 0$$

fixes the admissible global closure form of the scalar-time potential.

The localized coherence background

$$\Theta_\infty$$

determines the asymptotic evaluation point governing the physical fluctuation operator.

The periodic ordering structure therefore depends not on the absolute value of:

$$V^{(3)}, \qquad V^{(4)},$$

individually, but on their closure-preserving relative hierarchy.

That hierarchy remains invariant under asymptotic scalar-time closure evaluation.

M Explicit Scalar-Time Inversion Threshold Window

We now compute the explicit scalar-time coherence thresholds governing the observed cross-shell inversion hierarchy.

M.1 General Threshold Structure

The inversion condition between two sectors

$$(n,\ell) \quad \text{and} \quad (n',\ell')$$

is

$$\beta > \Lambda_{(n,\ell)\to(n',\ell')},$$

where

$$\Lambda_{(n,\ell)\to(n',\ell')} = \frac{\frac{1}{n'^2} - \frac{1}{n^2}}{\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}}.$$

We now evaluate the experimentally relevant inversion thresholds explicitly.

M.2 The $4s \to 3d$ Threshold

For

$$4s: \qquad n = 4, \quad \ell = 0,$$

and

$$3d: \qquad n' = 3, \quad \ell' = 2,$$

the numerator becomes

$$\frac{1}{9} - \frac{1}{16} = \frac{7}{144}.$$

The denominator becomes

$$\frac{1}{32} - \frac{1}{67.5}.$$

Thus

$$\Lambda_{4s\to 3d} = \frac{\frac{7}{144}}{\frac{1}{32} - \frac{1}{67.5}}.$$

Evaluating numerically gives

$$\Lambda_{4s\to 3d} \approx 2.96.$$

M.3 The $5s \to 4d$ Threshold

For

$$5s: \qquad n = 5, \quad \ell = 0,$$

and

$$4d: \qquad n' = 4, \quad \ell' = 2,$$

the numerator becomes

$$\frac{1}{16} - \frac{1}{25} = \frac{9}{400}.$$

The denominator becomes

$$\frac{1}{62.5} - \frac{1}{160}.$$

Thus

$$\Lambda_{5s\to 4d} = \frac{\frac{9}{400}}{\frac{1}{62.5} - \frac{1}{160}}.$$

Evaluating numerically gives

$$\Lambda_{5s\to 4d} \approx 2.31.$$

M.4 The $6s \to 4f$ Threshold

For

$$6s: \qquad n = 6, \quad \ell = 0,$$

and

$$4f: \qquad n' = 4, \quad \ell' = 3,$$

the numerator becomes

$$\frac{1}{16} - \frac{1}{36} = \frac{5}{144}.$$

The denominator becomes

$$\frac{1}{108} - \frac{1}{224}.$$

Thus

$$\Lambda_{6s\to 4f} = \frac{\frac{5}{144}}{\frac{1}{108} - \frac{1}{224}}.$$

Evaluating numerically gives

$$\Lambda_{6s\to 4f} \approx 7.28.$$

M.5 Existence of a Nonempty Coherence Window

The experimentally observed inversion hierarchy requires:

$$\beta > 2.96,$$

$$\beta > 2.31,$$

and

$$\beta > 7.28.$$

Therefore the observed inversion structure admits the consistency window

$$\beta > 7.28.$$

A single scalar-time coherence-binding parameter therefore simultaneously reproduces:

$$4s < 3d,$$

$$5s < 4d,$$

and

$$6s < 4f.$$

M.6 Unified Global Coherence Window

Collecting the experimentally required inversion conditions gives:

$$\Lambda_{4s\to 3d} \approx 2.96,$$

$$\Lambda_{5s\to 4d} \approx 2.31,$$

$$\Lambda_{6s\to 4f} \approx 7.28.$$

Accordingly, the observed inversion hierarchy requires the single scalar-time coherence condition

$$\beta > 7.28.$$

Within this regime, the scalar-time ordering function

$$\mathcal{E}_{n\ell} = -\frac{1}{n^2} - \frac{\beta}{n^3(\ell + \frac{1}{2})}$$

simultaneously reproduces:

$$4s < 3d,$$

$$5s < 4d,$$

and

$$6s < 4f,$$

while preserving the intermediate ordering relations:

$$3d < 4p,$$

$$4d < 5p,$$

and

$$4f < 5d < 6p.$$

Thus the periodic ordering hierarchy emerges from a single globally admissible scalar-time coherence-binding regime rather than from independent empirical shell-ordering prescriptions.

The existence of a nonempty global coherence window is a nontrivial consistency condition on the TSFT periodicity derivation.

Table 2: Scalar-time inversion thresholds

Inversion	Threshold Condition	Numerical Value
$4s \to 3d$	$\beta > \Lambda_{4s\to 3d}$	2.96
$5s \to 4d$	$\beta > \Lambda_{5s\to 4d}$	2.31
$6s \to 4f$	$\beta > \Lambda_{6s\to 4f}$	7.28

M.7 Interpretive Consequence

The periodic inversion hierarchy is therefore not generated through independent empirical ordering rules.

Instead, the observed cross-shell structure emerges from a single scalar-time coherence-binding regime satisfying a unified inversion-threshold system.

The existence of a nonempty admissible coherence window is a nontrivial consistency condition on the scalar-time closure framework.

N Effective Sign Structure of the Coherence-Binding Sector

We now clarify the physical sign of the scalar-time coherence-binding contribution.

The purpose of the present appendix is to distinguish between the bare asymptotic operator residue and the effective observable stabilization term already defined in the prior TSFT β-closure analysis.

N.1 Bare Operator Residue

The asymptotic fluctuation operator contains the formal curvature expansion

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}).$$

If the term

$$+\frac{\beta}{r^2}$$

is treated in isolation as an ordinary first-order perturbation, then

$$\Delta\epsilon_{n\ell}^{\text{formal}} = +\frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

This is the bare spectral residue of the asymptotic operator.

It is not, by itself, the full scalar-time coherence-stabilization energy.

N.2 Prior TSFT β-Closure Result

The earlier TSFT periodicity closure analysis defined the physical β sector not as a repulsive centrifugal enhancement, but as the effective coherence-binding contribution associated with finite internal temporal allocation.

In that prior closure stage,

$$\beta = B\eta_{\text{int}}^2,$$

with

$$\eta_{\text{int}} > 0$$

for massive bound configurations and

$$\eta_{\text{int}} = 0$$

in the null propagative limit.

The corresponding effective scalar-time ordering spectrum was therefore defined as

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

Thus the negative sign is not chosen ad hoc in the present paper. It is inherited from the previously established TSFT coherence-binding convention.

N.3 Coherence-Stabilization Interpretation

Finite internally retained temporal allocation strengthens localized coherence relative to the null propagation sector.

Because the correction scales as

$$\frac{1}{\ell+\frac{1}{2}},$$

the stabilization is strongest for

$$\ell = 0$$

and weakens for higher angular sectors.

Accordingly,

$$|\Delta\epsilon_{ns}| > |\Delta\epsilon_{np}| > |\Delta\epsilon_{nd}| > |\Delta\epsilon_{nf}|.$$

This produces the low-angular-momentum stabilization required for cross-shell inversions such as

$$4s < 3d, \qquad 5s < 4d, \qquad 6s < 4f.$$

N.4 Resolution of the Apparent Sign Tension

The apparent sign tension arises only if the bare asymptotic residue

$$+\frac{\beta}{r^2}$$

is identified directly with the full physical stabilization energy.

Within the TSFT progression spine, that identification is not made.

The bare residue determines the magnitude and angular dependence of the coherence deformation.

The effective observable ordering sign is fixed by the prior temporal-efficiency β-closure result, where internally retained coherence contributes attractive stabilization energy.

Thus the physically relevant spectrum is

$$\boxed{\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}}$$

with

$$\beta = B\eta_{\text{int}}^2 > 0.$$

N.5 Null-Sector Consistency

In the photon limit,

$$\eta_{\text{int}} = 0,$$

and therefore

$$\beta = 0.$$

The coherence-binding correction vanishes continuously:

$$\epsilon_{n\ell} \to -\frac{\kappa^2}{4n^2}.$$

Thus the attractive coherence-binding sector is present only for internally retained temporal structure and disappears in the null propagative limit.

This is precisely the behavior required by temporal-efficiency closure.

The scalar-time coefficient

$$\beta$$

is not introduced in the present work as an arbitrary perturbative residue.

Rather, within the prior TSFT -closure framework,

$$\beta = B\eta_{\text{int}}^2$$

was already defined as the effective coherence-binding stabilization coefficient associated with finite internally retained temporal allocation.

Accordingly, the asymptotic scalar-time operator determines:

the angular scaling structure

and

the inversion-threshold geometry,

while the physical sign of the ordering correction follows directly from the previously established coherence-binding interpretation of the sector itself.

No additional effective parameter distinct from

$$\beta$$

is introduced.

O Imported Spin-Sector Structure and SU(2) Closure Dependence

We now clarify the role of spin-sector doubling within the present periodicity derivation.

The purpose of the present appendix is to distinguish between:

structures derived internally in the present work,

and

structures inherited from prior stages of the TSFT progression spine.

O.1 Angular Degeneracy from Rotational Symmetry

The scalar-time fluctuation operator yields angular coherence sectors labeled by:

$$\ell = 0, 1, 2, \dots$$

with magnetic degeneracy:

$$2\ell + 1.$$

This degeneracy follows directly from rotational symmetry and spherical harmonic decomposition of the scalar-time fluctuation operator.

Accordingly, the purely orbital subshell capacities are:

$$1,\ 3,\ 5,\ 7, \dots$$

O.2 Spin-Sector Doubling

The observed periodic-table capacities require:

$$2,\ 6,\ 10,\ 14, \dots$$

These arise from an additional doubling:

$$2(2\ell + 1).$$

The present work does not independently derive this doubling from the scalar-time fluctuation operator alone.

Instead, the doubling is inherited from the previously derived TSFT spin-sector closure framework developed in the SU(2) spectral geometry sector of the TSFT progression spine.

O.3 Relation to Prior TSFT Spin Closure Work

Previous TSFT work established that admissible degenerate scalar-time closure sectors generate:

projective SU(2) geometry,

two-state measurement structure,

and

spin-sector doubling.

Accordingly, the present periodicity derivation imports the existence of two admissible fermionic spin coherence orientations:

$$s = \pm\frac{1}{2}.$$

The resulting shell capacities become:

$$G_n = 2n^2,$$

while subshell capacities become:

$$C_\ell = 2(2\ell + 1).$$

O.4 Consistency of the Dependency Structure

The present paper therefore derives:

principal-shell quantization,

subshell splitting,

cross-shell inversion,

and

periodic ordering

from scalar-time closure dynamics.

The spin-sector doubling required to reproduce observed block capacities is inherited explicitly from prior TSFT spin/SU(2) closure results.

Accordingly, no hidden spin-sector insertion occurs within the present derivation itself.

O.5 Interpretive Consequence

The periodic table depends jointly on:

scalar-time spectral closure,

rotational coherence degeneracy,

and

SU(2)-derived spin-sector doubling.

The present work therefore represents one stage within the broader TSFT closure progression rather than an isolated standalone derivation independent of the preceding spin-sector framework.

P Asymptotic Termination of Cross-Shell Inversions

We now derive the asymptotic behavior of the scalar-time inversion hierarchy.

The purpose of the present appendix is to show that cross-shell inversions occur only within a finite coherence-ordering regime and do not continue indefinitely at arbitrarily large shell index.

P.1 Scalar-Time Ordering Competition

The scalar-time spectral hierarchy is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}.$$

Cross-shell inversion occurs when the coherence-binding stabilization term overcomes the principal-shell separation between neighboring sectors.

The ordering competition is therefore governed by:

$$O(n^{-2})$$

versus

$$O(n^{-3}).$$

P.2 Relative Suppression of the Coherence-Binding Sector

Define the ratio:

$$R_{n\ell} = \frac{|\Delta\epsilon_{n\ell}|}{|\epsilon_n^{(0)}|}.$$

Substituting:

$$\Delta\epsilon_{n\ell} = -\frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})},$$

and

$$\epsilon_n^{(0)} = -\frac{\kappa^2}{4n^2},$$

gives

$$R_{n\ell} = \frac{\beta}{n(\ell+\frac{1}{2})}.$$

Thus:

$$R_{n\ell} \to 0 \qquad (n\to\infty).$$

The coherence-binding sector therefore becomes asymptotically negligible relative to the principal-shell hierarchy.

P.3 Suppression of Large-n Inversions

Consider neighboring competing sectors:

$$(n+1, s) \quad \text{and} \quad (n, d),$$

or more generally:

$$(n+1, \ell_1), \qquad (n, \ell_2).$$

The principal-shell separation scales approximately as

$$\Delta_n^{(0)} \sim \frac{1}{n^3}.$$

The coherence-binding correction difference scales as

$$\Delta_n^{(\beta)} \sim \frac{\beta}{n^3}\left(\frac{1}{\ell_1+\frac{1}{2}} - \frac{1}{\ell_2+\frac{1}{2}}\right).$$

However, the relative angular splitting factor is bounded:

$$\left|\frac{1}{\ell_1+\frac{1}{2}} - \frac{1}{\ell_2+\frac{1}{2}}\right| < 2.$$

Accordingly, the inversion-driving sector cannot grow indefinitely with shell index.

Beyond sufficiently large

$$n,$$

the principal-shell ordering dominates.

P.4 Existence of a Finite Inversion Regime

The scalar-time inversion hierarchy therefore admits only a finite inversion regime.

Cross-shell inversions occur only while:

$$\frac{\beta}{n(\ell+\frac{1}{2})}$$

remains sufficiently large relative to neighboring principal-shell separation.

At sufficiently large shell index,

$$\epsilon_{n\ell} \sim -\frac{\kappa^2}{4n^2},$$

and the ordering hierarchy approaches pure principal-shell structure.

The scalar-time framework therefore predicts:

finite inversion hierarchy,

rather than indefinitely cascading inversion complexity.

P.5 Comparison with Empirical Ordering Rules

The empirical Madelung rule provides no intrinsic mechanism governing the eventual termination of inversion structure.

Within TSFT, however, inversion suppression follows directly from asymptotic scaling:

$$n^{-3}$$

versus

$$n^{-2}.$$

The asymptotic restoration of principal-shell dominance is therefore a structural prediction of scalar-time closure geometry.

P.6 Predictive Consequence

The scalar-time framework predicts that sufficiently high-shell superheavy sectors should progressively approach principal-shell ordering:

$$ns < np < nd < nf$$

within fixed-shell organization.

Accordingly, deviations between TSFT ordering and naive Madelung continuation should become increasingly pronounced in sufficiently high-shell superheavy regimes.

This provides a potentially falsifiable prediction distinguishing scalar-time closure ordering from purely empirical shell-ordering prescriptions.

Q Friedrichs Extension and Self-Adjoint Completion of the Scalar-Time Operator

We now specify the self-adjoint completion of the scalar-time fluctuation operator.

The purpose of the present appendix is to identify the mathematically admissible self-adjoint realization governing the scalar-time spectral hierarchy.

Q.1 Scalar-Time Fluctuation Operator

The asymptotic scalar-time operator is

$$L_\Theta = -\nabla^2 - \frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}).$$

The leading singular behavior near the origin is governed by:

$$\frac{\beta}{r^2}.$$

Q.2 Lower-Boundedness

The inverse-square operator

$$-\nabla^2 + \frac{\beta}{r^2}$$

is bounded below provided:

$$\beta > -\frac{1}{4}.$$

Within TSFT,

$$\beta > 0,$$

since the coherence-binding sector is generated by finite internally retained temporal allocation.

Accordingly:

$$L_\Theta$$

remains lower bounded.

Q.3 Symmetric Dense Domain

Define the initial operator on the dense domain:

$$C_0^\infty(\mathbb{R}^3 \setminus \{0\}),$$

consisting of smooth compactly supported functions excluding the singular origin.

Integration by parts yields:

$$\langle u, L_\Theta v\rangle = \langle L_\Theta u, v\rangle,$$

so the scalar-time fluctuation operator is symmetric on the initial dense domain.

Q.4 Self-Adjoint Completion

Because:

$$L_\Theta$$

is symmetric and lower bounded, the operator admits a distinguished self-adjoint completion through the Friedrichs extension.

The Friedrichs extension preserves:

lower boundedness,

spectral stability,

and

finite-energy admissibility.

Accordingly, the scalar-time spectral hierarchy is defined through the Friedrichs self-adjoint realization:

$$L_\Theta^{(F)}.$$

Q.5 Limit-Point Structure

For sufficiently positive:

$$\beta,$$

the inverse-square sector approaches the limit-point regime near the origin.

In this regime, the admissible scalar-time spectral structure becomes uniquely determined without additional boundary-condition ambiguity.

Thus finite internally retained temporal allocation strengthens spectral regularization rather than destabilizing the operator.

Q.6 Interpretive Consequence

The periodic shell hierarchy therefore arises from a mathematically admissible self-adjoint scalar-time spectral operator.

The scalar-time ordering structure is not generated by a formal unstable expansion.

Instead, it emerges from the Friedrichs-completed spectral geometry of localized scalar-time coherence sectors.

R Non-Inversion Consistency Checks

We now verify that the scalar-time coherence-binding window producing the observed cross-shell inversions does not simultaneously generate widespread unphysical inversions.

The purpose of the present appendix is to test whether the same globally admissible coherence-binding regime:

$$\beta > 7.28$$

preserves the empirically stable ordering of sectors that are not observed to invert.

R.1 General Non-Inversion Condition

For two sectors:

$$(n, \ell), \qquad (n', \ell'),$$

the scalar-time ordering hierarchy is

$$\epsilon_{n\ell} = -\frac{1}{n^2} - \frac{\beta}{n^3(\ell + \frac{1}{2})}.$$

A non-inversion condition requires

$$\epsilon_{n\ell} < \epsilon_{n'\ell'}.$$

Substituting the ordering function gives

$$-\frac{1}{n^2} - \frac{\beta}{n^3(\ell + \frac{1}{2})} < -\frac{1}{n'^2} - \frac{\beta}{n'^3(\ell' + \frac{1}{2})}.$$

Rearranging,

$$\beta < \frac{\frac{1}{n'^2} - \frac{1}{n^2}}{\frac{1}{n^3(\ell + \frac{1}{2})} - \frac{1}{n'^3(\ell' + \frac{1}{2})}}.$$

Thus empirically stable non-inversions generate upper bounds on the admissible scalar-time coherence-binding regime.

R.2 Example: $3s$ Versus $2p$

Empirically,

$$2p < 3s.$$

Accordingly, the scalar-time ordering must satisfy

$$\epsilon_{2p} < \epsilon_{3s}.$$

Using:

$$(n, \ell) = (2, 1), \qquad (n', \ell') = (3, 0),$$

gives

$$\beta < \frac{\frac{1}{9} - \frac{1}{4}}{\frac{1}{8(1.5)} - \frac{1}{27(0.5)}}.$$

Evaluating,

$$\frac{1}{9} - \frac{1}{4} = -\frac{5}{36},$$

and

$$\frac{1}{12} - \frac{2}{27} = \frac{1}{108}.$$

Thus

$$\beta < 15.$$

Therefore the observed ordering:

$$2p < 3s$$

remains stable throughout the inversion-producing regime

$$7.28 < \beta < 15.$$

R.3 Example: $3p$ Versus $4s$

Empirically,

$$3p < 4s.$$

The scalar-time ordering condition becomes

$$\epsilon_{3p} < \epsilon_{4s}.$$

Using:

$$(n, \ell) = (3, 1), \qquad (n', \ell') = (4, 0),$$

gives

$$\beta < \frac{\frac{1}{16} - \frac{1}{9}}{\frac{1}{27(1.5)} - \frac{1}{64(0.5)}}.$$

Evaluating,

$$\frac{1}{16} - \frac{1}{9} = -\frac{7}{144},$$

and

$$\frac{2}{81} - \frac{1}{32} = -\frac{17}{2592}.$$

Thus

$$\beta < 7.41.$$

Accordingly,

$$3p < 4s$$

remains stable only near the lower edge of the admissible inversion regime.

This result reflects the experimentally known proximity of the

$$4s$$

sector to neighboring shell competition.

R.4 Interpretive Consequence

The scalar-time framework therefore does not generate unrestricted inversion proliferation.

Instead, the coherence-binding sector produces:

a finite admissible inversion window

bounded both below and above by competing ordering constraints.

Observed cross-shell inversions require:

$$\beta > 7.28,$$

while empirically stable lower-shell orderings impose upper constraints such as:

$$\beta < 15.$$

Thus the scalar-time ordering structure occupies a constrained intermediate coherence regime rather than an arbitrarily strong inversion sector.

R.5 Physical Interpretation

The coexistence of:

$$\text{inversion-producing}$$

and

$$\text{non-inverting}$$

constraints reflects the balance between:

$$\text{principal-shell quantization}$$

and

$$\text{coherence-binding stabilization.}$$

For sufficiently weak:

$$\beta,$$

no cross-shell inversions occur.

For sufficiently large:

$$\beta,$$

lower-shell stability begins to break down.

The observed periodic hierarchy therefore occupies a finite scalar-time coherence regime in which inversion structure emerges without destroying global shell organization.

R.6 Predictive Consequence

The existence of both lower and upper coherence-ordering bounds implies that sufficiently strong scalar-time coherence-binding sectors should eventually generate anomalous ordering behavior beyond the empirically observed periodic regime.

Accordingly, the scalar-time framework predicts that sufficiently extreme high-Z, relativistic, or superheavy sectors may exhibit ordering anomalies departing from both naive principal-shell hierarchy and empirical Madelung continuation. The narrowness of the admissible coherence-ordering window suggests that atomic periodicity may occupy a near-critical scalar-time closure regime separating insufficient inversion structure from excessive shell destabilization.