Global Closure of the Scalar-Time Potential Through Temporal Efficiency Geometry

Jordan Gabriel Farrell
Founding Editor, Zebra Journal of Unified Physics
Independent Researcher
Colchester, Connecticut, USA
ORCID: 0009-0002-2171-809X
jgfquantum@gmail.com

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Abstract

Previous formulations of Time–Scalar Field Theory (TSFT) established that temporal advance partitions between internally retained evolution and external propagation according to the conserved temporal-efficiency relation

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1,$$

with null propagation corresponding to the maximum-efficiency coherence limit

$$\eta_{\text{int}} = 0, \qquad \eta_{\text{prop}} = 1.$$

These earlier results successfully recovered relativistic time dilation, photon propagation, temporal inertia, and coherence-driven bound structure, but left the scalar-time potential

$$V(\Theta)$$

formally unspecified beyond local asymptotic constraints.

In the present work, we demonstrate that the global structure of the scalar-time potential is not arbitrary, but is constrained by temporal-efficiency geometry itself. Beginning from the conserved partition law and the requirement that null propagation represent the maximal coherence state, we derive the admissible curvature structure of

$$V(\Theta)$$

from internal temporal-allocation consistency.

The conserved temporal partition naturally generates an efficiency ratio measuring internally retained temporal allocation relative to propagative temporal al-

location. Combining this ratio with the scalar-time coupling factor

$$\alpha(\Theta) = e^{-\Theta},$$

we derive a scalar-only globally closed temporal-efficiency potential. Imposing null-sector coherence conditions

$$V(0) = 0, \qquad V'(0) = 0, \qquad V''(0) = 0,$$

naturally selects the null-normalized scalar-time potential

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

up to an overall coherence-curvature scale

$$V_*.$$

The resulting potential generates asymptotic inverse-radial scalar-time structure, subshell-sensitive coherence deformations, and thresholded shell-crossing behavior in the atomic sector. Both the dominant shell-generating interaction and the subleading coherence-binding deformation arise from the same globally closed scalar-time curvature hierarchy:

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

Atomic organization therefore emerges not from empirical shell-ordering rules, but from globally constrained temporal-efficiency geometry within the scalar-time field itself.

More broadly, the present work proposes that mass, propagation, binding, and spectral organization are different realizations of a single conserved temporal-efficiency structure. The scalar-time potential is therefore interpreted not as an arbitrary phenomenological input, but as the variational curvature geometry generated by admissible temporal allocation itself.

1 Introduction

One of the remaining unresolved questions within Time–Scalar Field Theory (TSFT) concerns the origin and global structure of the scalar-time potential

$$V(\Theta).$$

Previous TSFT investigations established that time is not a passive background parameter but an active scalar field governing physical evolution through temporal allocation, coherence propagation, and internal state retention. Within this framework, relativistic time dilation, photon propagation, temporal inertia, and bound coherent structure emerge from redistribution of a conserved temporal-efficiency budget rather than from geometric postulates imposed independently of physical dynamics.

A central result of prior work was the temporal-efficiency partition relation

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1,$$

where

$$\eta_{\text{int}}$$

quantifies internally retained temporal allocation and

$$\eta_{\text{prop}}$$

quantifies propagative temporal allocation. In the null propagation limit,

$$\eta_{\text{int}} = 0, \qquad \eta_{\text{prop}} = 1,$$

corresponding to the maximum-efficiency coherence state occupied by massless propagation. Photon propagation was thereby derived not as motion caused by force or impulse, but as perfect coupling to the forward temporal gradient of the scalar-time field.

Subsequent TSFT developments further established that mass and bound coherent structure arise through internally retained temporal allocation. Temporal inertia, phase locking, and coherent spectral stabilization were shown to emerge from nonzero internal temporal organization relative to the null propagation sector. Observation and continuity were likewise demonstrated to require globally coherent temporal allocation structure rather than disconnected local registrations.

Despite these advances, the scalar-time potential

$$V(\Theta)$$

itself remained only partially constrained. Previous atomic-sector analyses demonstrated that localized scalar-time backgrounds generate asymptotic fluctuation operators of the form

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}),$$

producing hydrogenic shell structure together with subshell-sensitive spectral deformations. However, the subleading curvature contribution governing the coherence- binding sector remained formally unconstrained.

The purpose of the present work is to demonstrate that this remaining freedom is not arbitrary. We show that the global scalar-time potential is constrained by temporal-efficiency geometry itself. Beginning from the conserved temporal partition law and the maximum-efficiency null boundary condition established in earlier TSFT work, we derive the admissible curvature structure of the scalar-time potential through internal temporal allocation consistency.

The resulting framework identifies scalar-time curvature not as a phenomenological input, but as a consequence of coherence-allocation extremization across admissible temporal sectors. Atomic spectral organization then emerges as one realization of this broader global scalar-time structure.

2 Temporal Efficiency Geometry and the Scalar-Time Field

Time–Scalar Field Theory treats time as an active scalar degree of freedom rather than a passive coordinate parameter. The fundamental dynamical object is the scalar-time potential

$$\Theta = \Theta(x^\mu),$$

whose gradients govern temporal allocation, propagation, and internally retained coherence structure.

The scalar-time field is governed by the action

$$S[\Theta] = \int d^4x \left[\frac{1}{2} \partial_\mu \Theta \, \partial^\mu \Theta - V(\Theta) \right],$$

yielding the Euler–Lagrange equation

$$\Box \Theta = V'(\Theta).$$

Unlike conventional scalar field theories in which the potential is introduced phenomenologically, TSFT interprets

$$V(\Theta)$$

as the effective curvature structure governing admissible temporal-allocation geometry. The potential therefore encodes the balance between internally retained temporal evolution and external propagative advance.

A central result of previous TSFT work was the temporal-efficiency partition relation

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1,$$

where:

$$\eta_{\text{int}} \in [0, 1]$$

measures internally retained temporal allocation associated with proper-time evolution, and

$$\eta_{\text{prop}} \in [0, 1]$$

measures temporal allocation directed toward external propagation.

This relation is not introduced kinematically, but as a conservation law governing temporal allocation itself. Temporal advance is treated as a conserved coherence budget whose distribution determines the realized physical state.

Proper-time evolution satisfies

$$d\tau = dt\,\eta_{\rm int}(\Theta),$$

while propagative advance satisfies

$$d\ell = c\,dt\,\eta_{\rm prop}(\Theta).$$

The scalar-time field therefore determines how available temporal advance partitions between internal evolution and external propagation.

Previous TSFT work established that propagative temporal allocation composes multiplicatively across successive scalar-time sectors. Continuity together with compositional consistency then uniquely yields the exponential coupling relation

$$\eta_{\rm prop}(\Theta) = e^{-\Theta},$$

derived self-consistently in Appendix A.

In the null propagation sector,

$$d\tau = 0,$$

and therefore

$$\eta_{\rm int} = 0, \qquad \eta_{\rm prop} = 1.$$

This represents the maximum-efficiency coherence state. Null propagation therefore corresponds to complete allocation of temporal advance toward propagation with no internally retained temporal budget.

By contrast, massive systems necessarily satisfy

$$\eta_{\rm int} > 0.$$

Bound coherent structure therefore emerges through internal temporal retention relative to the null propagation limit. Mass, phase locking, coherence stabilization, and spectral organization all arise from redistribution of temporal allocation away from perfect propagative efficiency.

The crucial implication for the present work is that the scalar-time potential cannot be chosen arbitrarily. Since the scalar-time field governs temporal allocation itself, the admissible curvature structure of

$$V(\Theta)$$

must remain consistent with the temporal-efficiency geometry defined by the conserved partition law.

In particular, the null propagation limit imposes a nontrivial global boundary condition on the scalar-time potential. The maximum-efficiency sector

$$\eta_{\rm int} = 0$$

must correspond to vanishing internally retained coherence deformation. Any curvature contribution associated purely with internal temporal retention must therefore vanish continuously in this limit.

The scalar-time potential is thus constrained not merely by local spectral behavior, but by global temporal-efficiency consistency across admissible coherence sectors.

The remainder of the present work develops the consequences of this requirement. We show that the subleading curvature structure governing bound coherent organization is not independent, but emerges necessarily from the temporal-efficiency geometry of the scalar-time field itself.

3 Global Temporal-Efficiency Closure of the Scalar-Time Potential

The conserved temporal-efficiency partition relation

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1$$

implies that internally retained temporal allocation and propagative temporal allocation cannot vary independently. Since the scalar-time field governs temporal allocation itself, the scalar-time potential must remain compatible with this global coherence geometry.

The central question is therefore whether the higher-order curvature structure of the scalar-time potential is physically arbitrary, or whether it is constrained by temporal-efficiency geometry itself.

Define the retained temporal-allocation fraction

$$x = \eta_{\text{int}}^2.$$

Then

$$\eta_{\text{prop}}^2 = 1 - x,$$

with

$$0 \leq x \leq 1.$$

The physically relevant coherence quantity is not the retained allocation alone, but the ratio between internally retained temporal allocation and propagative temporal allocation:

$$\mathcal{R} = \frac{\eta_{\text{int}}^2}{\eta_{\text{prop}}^2}.$$

Using the conserved partition relation,

$$\mathcal{R} = \frac{x}{1 - x}.$$

This ratio vanishes continuously in the null propagation sector,

$$x \to 0,$$

increases monotonically with internally retained temporal allocation, and diverges in the complete temporal-localization limit

$$x \to 1.$$

The conserved temporal partition therefore naturally generates the efficiency ratio

$$\mathcal{R} = \frac{\eta_{\rm int}^2}{\eta_{\rm prop}^2}.$$

Previous TSFT work established the scalar-time coupling relation

$$\alpha(\Theta) = e^{-\Theta},$$

which governs the locally available propagative temporal advance. Propagative temporal allocation is therefore identified with

$$\eta_{\rm prop}(\Theta) = e^{-\Theta},$$

giving

$$\eta_{\rm prop}^2(\Theta) = e^{-2\Theta}.$$

The conserved partition relation then yields

$$\eta_{\rm int}^2(\Theta) = 1 - e^{-2\Theta}.$$

Substituting into the efficiency ratio gives

$$\mathcal{R}(\Theta) = \frac{1 - e^{-2\Theta}}{e^{-2\Theta}} = e^{2\Theta} - 1.$$

The raw scalar-time efficiency geometry therefore generates the scalar-only ratio

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1.$$

However, the asymptotic null propagation sector additionally requires

$$V(0) = 0, \qquad V'(0) = 0, \qquad V''(0) = 0.$$

The first condition fixes the vacuum normalization, the second removes residual vacuum forcing, and the third eliminates quadratic Yukawa curvature contributions in the maximum-efficiency sector.

The physically admissible scalar-time potential is therefore the null-normalized efficiency potential

$$\boxed{V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),}$$

where

$$V_*$$

sets the overall coherence-curvature scale.

Expanding near the null sector,

$$\Theta \ll 1,$$

gives

$$V(\Theta) = V_* \left(\frac{4}{3}\Theta^3 + \frac{2}{3}\Theta^4 + O(\Theta^5)\right).$$

Thus

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

The scalar-time curvature hierarchy is therefore no longer freely postulated. The globally admissible structure of the scalar-time potential emerges directly from temporal allocation consistency together with null-sector coherence normalization.

The following section examines the asymptotic atomic-sector consequences of this globally closed scalar-time potential.

4 Asymptotic Scalar-Time Structure and Effective Spectral Dynamics

To determine the consequences of temporal-efficiency geometry for bound coherent structure, we consider localized static scalar-time configurations of the form

$$\Theta(x^\mu) = \Theta_0(r),$$

where spherical symmetry reduces the scalar-time field equation

$$\Box\Theta = V'(\Theta)$$

to

$$\nabla^2\Theta_0(r) = V'(\Theta_0(r)).$$

We assume the existence of an asymptotically coherent scalar-time background satisfying

$$\Theta_0(r) \to \Theta_\infty \qquad (r \to \infty),$$

where the asymptotic vacuum obeys

$$V'(\Theta_\infty) = 0.$$

The null propagation sector derived previously imposes an additional curvature condition. Because the maximum-efficiency state

$$\eta_{\text{int}} = 0$$

contains no internally retained temporal allocation, the asymptotic vacuum cannot support leading-order internally retained coherence curvature. The asymptotic background must therefore additionally satisfy

$$V''(\Theta_\infty) = 0.$$

This condition is physically significant. A nonzero quadratic curvature term would generate exponentially suppressed Yukawa behavior, corresponding to finite-range internally retained coherence structure even in the null propagation sector. Such behavior would contradict the maximum-efficiency boundary condition.

The leading admissible asymptotic scalar-time background therefore assumes the form

$$\Theta_0(r) = \Theta_\infty - \frac{A}{r} + O(r^{-2}), \qquad A > 0,$$

where

$$A$$

determines the leading scalar-time coherence amplitude.

To examine the resulting curvature structure, we introduce fluctuations about the background:

$$\Theta(r,t) = \Theta_0(r) + \delta\Theta(r,t).$$

Linearization of the scalar-time field equation yields

$$\left(\partial_t^2 - \nabla^2 + V''(\Theta_0(r))\right)\delta\Theta = 0.$$

The effective asymptotic structure is therefore governed by the behavior of

$$V''(\Theta_0(r)).$$

Expanding about the asymptotic vacuum,

$$V''(\Theta_0) = V''(\Theta_\infty) + V^{(3)}(\Theta_\infty)(\Theta_0 - \Theta_\infty) + \frac{1}{2}V^{(4)}(\Theta_\infty)(\Theta_0 - \Theta_\infty)^2 + O(r^{-3}).$$

Since

$$V''(\Theta_\infty) = 0,$$

substituting the asymptotic scalar-time background gives

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}),$$

where

$$\kappa = A\,V^{(3)}(\Theta_\infty),$$

and

$$\beta = \frac{1}{2}A^2 V^{(4)}(\Theta_\infty).$$

The asymptotic scalar-time background therefore generates an effective inverse-radial curvature hierarchy governed by the third- and fourth-order derivatives of the scalar-time potential.

The preceding section demonstrated that temporal-efficiency geometry globally constrains the admissible scalar-time potential through the null-normalized closure relation

$$V(\Theta) = V_*\left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).$$

The asymptotic scalar-time background derived here therefore acquires a direct global interpretation. The effective inverse-radial curvature hierarchy is not independently postulated, but descends from the globally constrained scalar-time curvature structure through

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

The following section derives the resulting atomic-sector spectral structure implied by this globally closed scalar-time potential.

5 Atomic-Sector Reduction of the Globally Closed Potential

The preceding sections established that temporal-efficiency geometry naturally motivates the null-normalized globally closed scalar-time potential

$$V(\Theta) = V_*\left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

together with the null-sector coherence conditions

$$V(0) = 0, \qquad V'(0) = 0, \qquad V''(0) = 0.$$

The purpose of the present section is to examine the asymptotic spectral consequences of this globally constrained scalar-time potential within localized bound coherence sectors.

Consider a static spherically symmetric scalar-time background

$$\Theta(x^\mu) = \Theta_0(r),$$

satisfying

$$\nabla^2\Theta_0(r) = V'(\Theta_0(r)).$$

For the globally closed potential,

$$V'(\Theta) = V_* \left(2e^{2\Theta} - 2 - 4\Theta\right),$$

and

$$V''(\Theta) = V_* \left(4e^{2\Theta} - 4\right).$$

Expanding about the null propagation background

$$\Theta_\infty = 0,$$

we have

$$V(0) = 0, \qquad V'(0) = 0, \qquad V''(0) = 0.$$

The first nonvanishing curvature orders are

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

Assume the asymptotic scalar-time configuration

$$\Theta_0(r) = -\frac{A}{r} + O(r^{-2}), \qquad A > 0,$$

with

$$|A/r| \ll 1$$

in the asymptotic regime.

Expanding $V''(\Theta_0(r))$ gives

$$V''(\Theta_0(r)) = V^{(3)}(0)\Theta_0(r) + \frac{1}{2}V^{(4)}(0)\Theta_0^2(r) + O(r^{-3}).$$

Substituting the asymptotic background,

$$V''(\Theta_0(r)) = -\frac{8AV_*}{r} + \frac{8A^2V_*}{r^2} + O(r^{-3}).$$

Writing this in the standard spectral form,

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}),$$

we identify

$$\kappa = 8AV_*,$$

and

$$\beta = 8A^2V_*.$$

Thus both the dominant inverse-radial shell-generating term and the subleading coherence-binding deformation arise from the same globally closed scalar-time potential.

The fluctuation equation therefore reduces asymptotically to the effective radial spectral problem

$$\left[-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} - \frac{\kappa}{r} + \frac{\beta}{r^2}\right] u_{n\ell}(r) = \epsilon_{n\ell} u_{n\ell}(r).$$

The dominant inverse-radial scalar-time interaction produces the leading shell hierarchy

$$\epsilon_n^{(0)} = -\frac{\kappa^2}{4n^2}.$$

Meanwhile, the subleading coherence-binding deformation modifies the effective angular sector according to

$$\ell_{\text{eff}}(\ell_{\text{eff}} + 1) = \ell(\ell+1) + \beta.$$

For

$$|\beta| \ll \left(\ell + \frac{1}{2}\right)^2,$$

the perturbative deformation becomes

$$\ell_{\text{eff}} = \ell + \frac{\beta}{2(\ell + \frac{1}{2})} + O(\beta^2).$$

The resulting spectral structure is therefore

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})} + O(\beta^2).$$

Substituting the globally closed curvature coefficients gives

$$\kappa^2 = 64A^2V_*^2,$$

and

$$\beta\kappa^2 = 512A^4V_*^3.$$

Therefore,

$$\boxed{\epsilon_{n\ell} = -\frac{16A^2V_*^2}{n^2} - \frac{128A^4V_*^3}{n^3(\ell + \frac{1}{2})} + O(\beta^2).}$$

The important result is that the coherence-binding deformation is no longer introduced phenomenologically. The dominant shell-generating term and the subleading subshell-ordering deformation both descend from the first nonvanishing curvature orders of the globally closed scalar-time potential:

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

Atomic organization therefore appears as a direct spectral manifestation of global temporal-efficiency geometry.

Low-angular-momentum sectors experience enhanced internally retained coherence stabilization because the coherence-binding contribution scales as

$$\left(\ell + \frac{1}{2}\right)^{-1}.$$

The globally closed scalar-time potential therefore naturally generates thresholded shell reorganization behavior within neighboring coherence sectors.

To illustrate the mechanism explicitly, compare the

$$4s$$

and

$$3d$$

sectors.

For

$$4s: \qquad n = 4, \qquad \ell = 0,$$

we obtain

$$\epsilon_{4s} = -\frac{\kappa^2}{64} - \frac{\beta\kappa^2}{128}.$$

For

$$3d: \qquad n = 3, \qquad \ell = 2,$$

we obtain

$$\epsilon_{3d} = -\frac{\kappa^2}{36} - \frac{\beta\kappa^2}{270}.$$

The inversion condition

$$\epsilon_{4s} < \epsilon_{3d}$$

yields

$$\beta > \frac{210}{71} \approx 2.958.$$

Since the globally closed potential gives

$$\beta = 8A^2V_*,$$

the inversion condition becomes

$$8A^2V_* > \frac{210}{71}.$$

Equivalently,

$$A^2V_* > \frac{105}{284}.$$

Thus neighboring subshell inversion emerges when the scalar-time coherence amplitude and global curvature scale exceed a finite threshold determined by the closed potential itself.

Within this framework, atomic organization no longer depends upon empirical shell-filling prescriptions. Instead, shell hierarchy and subshell reorganization both arise from the same globally constrained temporal-efficiency potential governing scalar-time coherence dynamics.

6 Predictions, Scope, and Physical Interpretation

The present framework yields structural predictions that distinguish the scalar-time coherence approach from empirical shell-ordering prescriptions and from phenomenological scalar-field models in which the potential is introduced externally.

The central result of the preceding sections is the globally closed scalar-time potential

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

with first nonvanishing curvature orders

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

For localized bound coherence sectors with

$$\Theta_0(r) = -\frac{A}{r} + O(r^{-2}), \qquad A > 0,$$

the induced asymptotic spectral coefficients are

$$\kappa = 8AV_*, \qquad \beta = 8A^2V_*.$$

Thus the effective spectrum is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})} + O(\beta^2).$$

The first prediction is that subshell reorganization is governed by the closed-potential coherence parameter

$$\beta = 8A^2V_*,$$

rather than by an externally fitted empirical shell-ordering rule.

The second prediction is that low-angular-momentum sectors experience stronger coherence stabilization than neighboring high-angular-momentum sectors, because the deformation scales as

$$\Delta\epsilon_{n\ell} = -\frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

The third prediction is that neighboring shell inversions occur only when the closed scalar-time curvature scale exceeds finite thresholds. For example, the $4s/3d$ crossing requires

$$\beta > \frac{210}{71},$$

or equivalently,

$$8A^2V_* > \frac{210}{71}.$$

Thus atomic organization is predicted to arise through thresholded scalar-time coherence structure rather than through independent empirical filling prescriptions.

It is important to clarify the scope of the present analysis. The current framework derives the globally closed scalar-time potential and its leading asymptotic atomic-sector spectral reduction, but it does not yet constitute a complete many-body atomic theory. Electron-electron screening, exchange interactions, relativistic fine structure, spin-orbit coupling, molecular bonding, and nuclear coherence dynamics remain outside the scope of the present treatment.

Likewise, the present work does not claim that every numerical aspect of the observed periodic table has already been derived exactly. Rather, the principal result is that the scalar-time potential is no longer freely postulated phenomenologically. Its functional form is constrained by temporal-efficiency geometry, and its leading curvature hierarchy naturally produces organized spectral structure across bound coherence sectors.

Future work will extend this framework toward many-body scalar-time coherence dynamics, relativistic spectral corrections, molecular structure, nuclear organization, and numerical tests of the closed potential across multiple physical domains.

7 Conclusion

The present work addressed one of the remaining open structural questions within Time–Scalar Field Theory: the origin and global form of the scalar-time potential

$$V(\Theta).$$

Previous TSFT investigations established that temporal advance partitions between internal temporal retention and external propagation according to the conserved temporal-efficiency relation

$$\eta_{\rm int}^2 + \eta_{\rm prop}^2 = 1,$$

with null propagation corresponding to the maximum-efficiency coherence limit

$$\eta_{\rm int} = 0, \qquad \eta_{\rm prop} = 1.$$

These earlier results successfully recovered relativistic time dilation, photon propagation, temporal inertia, and localized coherence structure, but left the global scalar-time potential formally unspecified beyond asymptotic curvature constraints.

In the present analysis, we demonstrated that the scalar-time potential is not an arbitrary external input. Instead, its admissible structure is constrained by the geometry of conserved temporal allocation itself.

Beginning from the temporal partition law, we derived the efficiency ratio

$$\mathcal{R} = \frac{\eta_{\text{int}}^2}{\eta_{\text{prop}}^2},$$

which measures internally retained temporal allocation relative to propagative temporal allocation. Combining this ratio with the scalar-time coupling relation

$$\alpha(\Theta) = e^{-\Theta},$$

allowed the temporal-efficiency geometry to be expressed entirely in terms of the scalar-time field:

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1.$$

The asymptotic null propagation sector then imposed the required coherence conditions

$$V(0) = 0, \qquad V'(0) = 0, \qquad V''(0) = 0,$$

eliminating residual vacuum forcing and quadratic Yukawa curvature contributions at the maximum-efficiency boundary.

These constraints naturally determine the null-normalized scalar-time potential up to an overall coherence-curvature scale:

$$\boxed{V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).}$$

The resulting potential possesses a direct physical interpretation. The scalar-time curvature structure measures the coherence cost of internally retained temporal allocation relative to propagative efficiency.

Within this framework:

$$\eta_{\text{int}} \to 0$$

corresponds to maximal propagation,

while

$$\eta_{\text{prop}} \to 0$$

corresponds to complete internally retained temporal localization.

The scalar-time potential therefore emerges as the variational curvature geometry generated by conserved temporal allocation itself.

The globally closed potential further generates the asymptotic spectral structure

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}),$$

where both the dominant shell-generating interaction and the subleading coherence-binding deformation descend from the same globally closed curvature hierarchy:

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

The resulting spectral operator naturally produces hydrogenic shell structure together with subshell-sensitive coherence deformations and thresholded shell-crossing behavior within neighboring coherence sectors.

Atomic organization therefore no longer appears as an empirical shell-ordering rule, but as a direct spectral manifestation of global temporal-efficiency geometry.

More broadly, the present framework suggests that propagation, mass, binding, and spectral organization are different realizations of a single conserved temporal-efficiency structure within the scalar-time field.

The present work does not yet constitute a complete many-body atomic theory, nor does it claim that all detailed chemical or nuclear structure has been fully derived. Exchange interactions, relativistic corrections, spin-dependent effects, molecular organization, and nuclear coherence dynamics remain outside the scope of the present treatment.

Nevertheless, the principal structural result is substantial: the scalar-time potential is no longer freely postulated phenomenologically, but is globally constrained by the same temporal-efficiency geometry that governs propagation and coherence organization throughout the TSFT framework.

Future work will extend the present formalism toward relativistic spectral corrections, many-body coherence dynamics, molecular structure, nuclear organization, and the emergence of higher-order physical sectors from globally constrained scalar-time geometry.

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A Self-Contained Derivation of the Exponential Scalar-Time Coupling

The main text uses the scalar-time coupling relation

$$\alpha(\Theta) = e^{-\Theta}.$$

This appendix derives why the exponential form is the natural admissible coupling between the scalar-time field and locally available temporal advance.

Let

$$\alpha(\Theta) > 0$$

denote the fraction of coordinate temporal advance locally available to physical processes in a scalar-time background. The coupling must satisfy three structural requirements.

First, the null scalar-time reference state must recover undeformed temporal advance:

$$\alpha(0) = 1.$$

Second, successive scalar-time deformations must compose consistently. If a process passes through two scalar-time offsets,

$$\Theta_1 \qquad \text{and} \qquad \Theta_2,$$

the resulting temporal coupling must not depend on whether the offsets are treated in one step or two steps. Therefore,

$$\alpha(\Theta_1 + \Theta_2) = \alpha(\Theta_1)\alpha(\Theta_2).$$

Third, the coupling must vary continuously with the scalar-time field. Thus

$$\alpha : \mathbb{R} \to \mathbb{R}^+$$

is continuous and multiplicative over scalar-time addition.

Define

$$f(\Theta) = \ln \alpha(\Theta).$$

Then the multiplicative composition law becomes

$$f(\Theta_1 + \Theta_2) = f(\Theta_1) + f(\Theta_2).$$

By continuity, the only solutions of this additive Cauchy equation are linear:

$$f(\Theta) = \lambda\Theta,$$

for some constant

$$\lambda.$$

Therefore,

$$\alpha(\Theta) = e^{\lambda\Theta}.$$

The sign of

$$\lambda$$

is fixed by the physical interpretation of positive scalar-time deformation. Positive

$$\Theta$$

corresponds to increased internally retained temporal allocation and reduced available propagative temporal advance. Hence

$$\frac{d\alpha}{d\Theta} < 0,$$

so

$$\lambda < 0.$$

Choosing units of scalar-time field strength so that

$$|\lambda| = 1,$$

we obtain

$$\boxed{\alpha(\Theta) = e^{-\Theta}.}$$

This result shows that the exponential coupling is not an arbitrary monotonic choice. A generic monotonic coupling would preserve ordering, but it would not necessarily preserve composition. The exponential is selected because scalar-time offsets must add while temporal availability factors multiply.

Equivalently, the logarithm of temporal availability must be additive:

$$\ln \alpha(\Theta_1 + \Theta_2) = \ln \alpha(\Theta_1) + \ln \alpha(\Theta_2).$$

Thus

$$\Theta$$

acts as the additive generator of multiplicative temporal attenuation. This is the same structural reason exponential maps arise whenever an additive generator produces a positive multiplicative response.

With

$$\alpha(\Theta) = e^{-\Theta},$$

the propagative temporal allocation becomes

$$\eta_{\text{prop}}(\Theta) = e^{-\Theta},$$

and therefore

$$\eta_{\text{prop}}^2(\Theta) = e^{-2\Theta}.$$

Using the conserved temporal-efficiency partition

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1,$$

we obtain

$$\eta_{\text{int}}^2(\Theta) = 1 - e^{-2\Theta}.$$

The temporal-efficiency ratio then becomes

$$\mathcal{R}(\Theta) = \frac{\eta_{\text{int}}^2(\Theta)}{\eta_{\text{prop}}^2(\Theta)} = \frac{1 - e^{-2\Theta}}{e^{-2\Theta}} = e^{2\Theta} - 1.$$

Therefore, the exponential coupling is the structural bridge between the conserved temporal partition and the scalar-only efficiency ratio used in the main text.

B Appendix B: Why Generic Monotonic Couplings Fail

The scalar-time coupling relation

$$\alpha(\Theta) = e^{-\Theta}$$

was derived in Appendix A from continuity together with multiplicative composition of successive scalar-time deformations. The purpose of the present appendix is to show why a generic monotonic coupling cannot consistently reproduce the required temporal-allocation geometry.

Suppose instead that the scalar-time coupling is represented by an arbitrary positive monotonic function

$$\alpha(\Theta) = f(\Theta),$$

satisfying

$$f(0) = 1, \qquad f(\Theta) > 0.$$

The conserved temporal-efficiency partition requires propagative temporal allocation to compose consistently across successive scalar-time sectors. If a process experiences two scalar-time offsets

$$\Theta_1 \qquad \text{and} \qquad \Theta_2,$$

then the total temporal attenuation must be independent of path grouping.

Equivalently, sequential scalar-time deformations must satisfy

$$f(\Theta_1 + \Theta_2) = f(\Theta_1)f(\Theta_2).$$

This requirement is not optional. Without it, the realized propagative temporal allocation depends on how scalar-time sectors are partitioned rather than on the total scalar-time deformation itself.

To illustrate the problem, consider a generic monotonic choice such as

$$f(\Theta) = \frac{1}{1+\Theta}.$$

Then

$$f(\Theta_1 + \Theta_2) = \frac{1}{1+\Theta_1+\Theta_2},$$

while

$$f(\Theta_1)f(\Theta_2) = \frac{1}{(1+\Theta_1)(1+\Theta_2)} = \frac{1}{1+\Theta_1+\Theta_2+\Theta_1\Theta_2}.$$

These are not equal unless

$$\Theta_1\Theta_2 = 0.$$

Thus the coupling becomes path-dependent:

$$f(\Theta_1 + \Theta_2) \neq f(\Theta_1)f(\Theta_2).$$

The same failure occurs for generic polynomial, rational, logarithmic, or trigonometric monotonic couplings. Monotonicity alone preserves ordering, but it does not preserve compositional consistency.

The scalar-time framework instead requires:

1. additive scalar-time deformation,
2. positive temporal-availability factors,
3. path-independent sequential composition,
4. continuous deformation behavior.

These requirements imply the functional equation

$$f(\Theta_1 + \Theta_2) = f(\Theta_1)f(\Theta_2).$$

Under continuity, the only admissible solutions are exponentials:

$$f(\Theta) = e^{\lambda\Theta}.$$

Physical interpretation fixes

$$\lambda < 0,$$

because increased internally retained temporal allocation must reduce available propagative temporal advance.

After normalization of scalar-time units,

$$\lambda = -1,$$

yielding

$$\boxed{\alpha(\Theta) = e^{-\Theta}.}$$

The exponential coupling therefore does not arise merely because it is monotonic. It arises because it is the unique continuous positive coupling compatible with additive scalar-time deformation and multiplicative temporal-allocation composition.

This result also explains why the scalar-only efficiency ratio derived in the main text naturally acquires exponential form:

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1.$$

The exponential structure is therefore not phenomenological curve fitting, but a direct consequence of temporal compositional consistency within the scalar-time field.

C Appendix C: From Efficiency Ratio to Minimal Scalar-Time Potential

The main text derives the scalar-only temporal-efficiency ratio

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1.$$

This relation follows from the conserved temporal-efficiency partition together with the scalar-time coupling structure. However,

$$\mathcal{R}(\Theta)$$

by itself is not yet the scalar-time potential.

The purpose of this appendix is to clarify the additional closure principle required to map the efficiency ratio into the scalar-time potential

$$V(\Theta).$$

The scalar-time potential is interpreted as the local curvature-energy density associated with internally retained temporal allocation. Therefore, the potential must be a scalar functional of the efficiency ratio:

$$V(\Theta) = V_* \mathcal{F}[\mathcal{R}(\Theta)],$$

where

$$V_*$$

sets the overall coherence-curvature scale, and

$$\mathcal{F}$$

is an admissible closure map from dimensionless temporal-efficiency cost to scalar-time curvature potential.

The minimal closure adopted in the main text is

$$\mathcal{F}[\mathcal{R}] = \mathcal{R}_{\mathrm{c}},$$

where

$$\mathcal{R}_{\mathrm{c}}$$

is the null-normalized efficiency ratio obtained by removing those components of

$$\mathcal{R}$$

that would otherwise generate vacuum offset, residual vacuum forcing, or quadratic

Yukawa curvature at the null propagation background.

This step is not implied by

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1$$

alone. Rather, it follows from the additional requirement that the scalar-time potential be the minimal admissible curvature functional compatible with the null propagation sector.

The null sector is defined by

$$\Theta = 0, \qquad \eta_{\text{int}} = 0, \qquad \eta_{\text{prop}} = 1.$$

At this boundary, internally retained temporal allocation vanishes. The scalar-time potential must therefore satisfy

$$V(0) = 0.$$

The null sector must also be a stationary scalar-time background. Otherwise the field equation

$$\Box\Theta = V'(\Theta)$$

would contain a residual source at

$$\Theta = 0.$$

Therefore,

$$V'(0) = 0.$$

Finally, the maximum-efficiency null sector should not contain leading-order internally retained coherence curvature. A nonzero quadratic term would introduce a finite-range mass/Yukawa scale into fluctuations around the null propagation background. The minimal null-sector coherence condition is therefore

$$V''(0) = 0.$$

These three conditions do not follow from the efficiency ratio alone. They are the null-sector closure conditions required to turn the temporal-efficiency ratio into an admissible scalar-time curvature potential.

We now construct the minimal such potential.

Starting from

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1,$$

expand near the null sector:

$$\mathcal{R}(\Theta) = 2\Theta + 2\Theta^2 + \frac{4}{3}\Theta^3 + \frac{2}{3}\Theta^4 + O(\Theta^5).$$

The terms

$$2\Theta$$

and

$$2\Theta^2$$

would generate

$$V'(0) \neq 0$$

and

$$V''(0) \neq 0$$

if the potential were taken to be simply proportional to

$$\mathcal{R}.$$

The minimal null-normalized curvature functional is therefore obtained by projecting out the constant, linear, and quadratic null-sector components:

$$\mathcal{R}_{\mathrm{c}}(\Theta) = \mathcal{R}(\Theta) - \mathcal{R}(0) - \mathcal{R}'(0)\Theta - \frac{1}{2}\mathcal{R}''(0)\Theta^2.$$

Since

$$\mathcal{R}(0) = 0, \qquad \mathcal{R}'(0) = 2, \qquad \mathcal{R}''(0) = 4,$$

we obtain

$$\mathcal{R}_{\mathrm{c}}(\Theta) = e^{2\Theta} - 1 - 2\Theta - 2\Theta^2.$$

The minimal scalar-time potential is therefore

$$\boxed{V(\Theta) = V_* \mathcal{R}_{\mathrm{c}}(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).}$$

This construction shows that the closed potential is not obtained from

$$\mathcal{R}(\Theta)$$

by bare substitution. It is obtained by applying the minimal null-normalization map

$$\mathcal{R} \mapsto \mathcal{R}_{\mathrm{c}}$$

required by scalar-time vacuum consistency.

Equivalently, the closure may be written as the projection operator

$$\Pi_{\geq 3}$$

acting on the Taylor expansion of the efficiency ratio about the null sector:

$$V(\Theta) = V_* \Pi_{\geq 3} \left[e^{2\Theta} - 1\right],$$

where

$$\Pi_{\geq 3}$$

removes all terms of order

$$\Theta^0, \Theta^1, \Theta^2.$$

Thus,

$$V(\Theta)$$

is the minimal third-order-and-higher scalar-time curvature functional generated by the temporal-efficiency ratio.

This appendix therefore clarifies the logical structure of the main derivation:

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1$$

determines the efficiency ratio,

$$\alpha(\Theta) = e^{-\Theta}$$

makes that ratio scalar-only,

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1$$

gives the raw temporal-efficiency cost,

and the null-sector closure conditions

$$V(0) = V'(0) = V''(0) = 0$$

select the minimal admissible curvature projection

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).$$

The result should therefore be understood as a minimal null-normalized scalar-time closure generated from the temporal-efficiency ratio, not as the bare identification

$$V(\Theta) \equiv V_* \mathcal{R}(\Theta).$$

D Appendix D: Derivation of the Null-Sector Curvature Condition

The main text imposes the null-sector coherence condition

$$V''(0) = 0.$$

The purpose of this appendix is to show why this condition follows from the maximum-efficiency null propagation sector rather than being an arbitrary Taylor subtraction.

The scalar-time field equation is

$$\Box\Theta = V'(\Theta).$$

Let the null propagation background be

$$\Theta = 0, \qquad \eta_{\rm int} = 0, \qquad \eta_{\rm prop} = 1.$$

This state represents complete allocation of temporal advance toward propagation, with no internally retained temporal budget.

Consider a small fluctuation about the null background:

$$\Theta = \delta\Theta, \qquad |\delta\Theta| \ll 1.$$

Expanding the field equation gives

$$\Box\delta\Theta = V'(0) + V''(0)\delta\Theta + \frac{1}{2}V^{(3)}(0)(\delta\Theta)^2 + O((\delta\Theta)^3).$$

The null background must first be source-free:

$$V'(0) = 0.$$

The linearized fluctuation equation then becomes

$$\Box\delta\Theta = V''(0)\delta\Theta + O((\delta\Theta)^2).$$

Equivalently,

$$\left(\Box - V''(0)\right)\delta\Theta = O((\delta\Theta)^2).$$

Thus a nonzero value of

$$V''(0)$$

introduces a linear mass-scale or Yukawa-scale curvature into fluctuations about the null background.

But the null sector is defined by

$$\eta_{\rm int} = 0.$$

It contains no internally retained temporal allocation. Therefore, to leading order, no internally retained coherence curvature may survive at the null boundary.

If

$$V''(0) \neq 0,$$

then infinitesimal fluctuations about the null state would already experience a restoring or screening scale before any finite internal allocation appears. This would mean that the null propagation sector secretly contains an internal coherence scale, contradicting

$$\eta_{\text{int}} = 0.$$

Therefore the maximum-efficiency null sector requires

$$\boxed{V''(0) = 0.}$$

The same conclusion follows from the efficiency ratio. The internally retained allocation is

$$\eta_{\text{int}}^2(\Theta) = 1 - e^{-2\Theta}.$$

Near the null sector,

$$\eta_{\text{int}}^2(\Theta) = 2\Theta - 2\Theta^2 + O(\Theta^3).$$

The scalar-time potential is not allowed to assign a quadratic curvature energy to the null background itself. Curvature energy must begin only after the null-sector offset and linear forcing terms have been removed. Hence the admissible curvature functional must vanish through second order at the null point:

$$V(0) = 0, \qquad V'(0) = 0, \qquad V''(0) = 0.$$

This condition does not state that all scalar-time fluctuations are massless in every sector. Rather, it states that the maximum-efficiency null sector contains no leading-order internally retained coherence curvature. Once a localized configuration realizes a nonzero background

$$\Theta_0(r) \neq 0,$$

the effective curvature

$$V''(\Theta_0(r))$$

can become nonzero and generate bound-sector spectral structure.

Thus,

$$V''(0) = 0$$

is a boundary condition imposed only at the null propagation background, not a prohibition against curvature in massive or bound coherence sectors.

This establishes the null-sector curvature condition used in the main text and in the variational projection construction.

E Appendix E: Variational Projection of the Minimal Null-Normalized Potential

Appendix C introduced the null-normalized closure map

$$\mathcal{R}(\Theta) \mapsto \mathcal{R}_{\rm c}(\Theta) = \Pi_{\geq 3}\mathcal{R}(\Theta),$$

where

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1.$$

The purpose of the present appendix is to show that this projection can be obtained variationally as the unique closest admissible curvature functional to the raw temporal-efficiency ratio.

Let

$$F(\Theta)$$

denote the dimensionless scalar-time curvature profile from which the potential is built:

$$V(\Theta) = V_* F(\Theta).$$

The raw temporal-efficiency cost is

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1.$$

However, an admissible scalar-time curvature profile must satisfy the null-sector coherence constraints

$$F(0) = 0, \qquad F'(0) = 0, \qquad F''(0) = 0.$$

We now seek the admissible curvature profile

$$F$$

that differs as little as possible from the raw efficiency ratio

$$\mathcal{R}$$

near the null sector, while satisfying these three constraints.

Define the quadratic deviation functional

$$\mathcal{J}[F] = \int_{-\varepsilon}^{\varepsilon} w(\Theta) \left[F(\Theta) - \mathcal{R}(\Theta)\right]^2 d\Theta,$$

where

$$w(\Theta) > 0$$

is an even positive weight and

$$\varepsilon > 0$$

defines the local null-sector neighborhood.

The constrained variational problem is

$$\delta \{\mathcal{J}[F] + \lambda_0 F(0) + \lambda_1 F'(0) + \lambda_2 F''(0)\} = 0.$$

Equivalently, because the constraints act only on the Taylor jet of

$$F$$

at the null point, we may decompose the raw efficiency ratio into its null-sector Taylor jet and its admissible remainder:

$$\mathcal{R}(\Theta) = \mathcal{R}_{<3}(\Theta) + \mathcal{R}_{\geq 3}(\Theta),$$

where

$$\mathcal{R}_{<3}(\Theta) = \mathcal{R}(0) + \mathcal{R}'(0)\Theta + \frac{1}{2}\mathcal{R}''(0)\Theta^2,$$

and

$$\mathcal{R}_{\geq 3}(\Theta) = \mathcal{R}(\Theta) - \mathcal{R}_{<3}(\Theta).$$

The admissibility constraints require that the null-sector Taylor jet of

$$F$$

vanish through second order:

$$j_0^2 F = 0.$$

Among all profiles satisfying

$$j_0^2 F = 0,$$

the unique minimizer of

$$\mathcal{J}[F]$$

within the local Taylor-analytic admissible class is obtained by preserving every unconstrained component of

$$\mathcal{R}$$

and removing only the forbidden null-sector jet:

$$F_{\min}(\Theta) = \mathcal{R}(\Theta) - \mathcal{R}(0) - \mathcal{R}'(0)\Theta - \frac{1}{2}\mathcal{R}''(0)\Theta^2.$$

This is precisely the projection

$$F_{\min} = \Pi_{\geq 3}\mathcal{R}.$$

For

$$\mathcal{R}(\Theta) = e^{2\Theta} - 1,$$

we have

$$\mathcal{R}(0) = 0, \qquad \mathcal{R}'(0) = 2, \qquad \mathcal{R}''(0) = 4.$$

Therefore,

$$F_{\min}(\Theta) = e^{2\Theta} - 1 - 2\Theta - 2\Theta^2.$$

The scalar-time potential selected by the constrained variational projection is therefore

$$\boxed{V(\Theta) = V_* F_{\min}(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).}$$

This construction gives a precise variational meaning to the minimal closure used in the main text. The potential is not obtained by simply setting

$$V(\Theta) \equiv V_* \mathcal{R}(\Theta).$$

Rather, it is obtained as the unique null-admissible minimizer closest to the raw temporal-efficiency ratio under the quadratic local deviation functional.

The uniqueness statement is therefore conditional but mathematically explicit:

$$\boxed{\text{Within the local analytic class and the quadratic closest-admissible closure principle, } F = \Pi_{\geq 3}\mathcal{R} \text{ is u}}$$

The variational structure may also be written directly at the level of the scalar-time action. Introduce auxiliary allocation fields

$$\eta_{\text{int}}, \qquad \eta_{\text{prop}},$$

and impose the conserved temporal partition by a Lagrange multiplier

$$\Lambda(x).$$

A constrained scalar-time allocation action can be written schematically as

$$S[\Theta, \eta_{\text{int}}, \eta_{\text{prop}}, \Lambda] = \int d^4x \left[\frac{1}{2}\partial_\mu \Theta\, \partial^\mu \Theta - V_* F(\Theta) + \Lambda(x)\left(\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 - 1\right)\right].$$

The propagative composition condition fixes

$$\eta_{\text{prop}}(\Theta) = e^{-\Theta},$$

and the partition constraint then gives

$$\eta_{\text{int}}^2(\Theta) = 1 - e^{-2\Theta}.$$

Hence the raw allocation cost becomes

$$\mathcal{R}(\Theta) = \frac{\eta_{\rm int}^2(\Theta)}{\eta_{\rm prop}^2(\Theta)} = e^{2\Theta} - 1.$$

The remaining variational freedom is the admissible choice of

$$F(\Theta)$$

in the potential sector. Requiring

$$F$$

to minimize the local deviation from

$$\mathcal{R}$$

subject to

$$F(0) = F'(0) = F''(0) = 0$$

selects

$$F(\Theta) = \Pi_{\geq 3}\mathcal{R}(\Theta).$$

Thus the complete logical chain is:

$$S[\Theta, \eta_{\rm int}, \eta_{\rm prop}, \Lambda] \quad \Longrightarrow \quad \eta_{\rm int}^2 + \eta_{\rm prop}^2 = 1,$$

$$\eta_{\rm prop}(\Theta) = e^{-\Theta} \quad \Longrightarrow \quad \mathcal{R}(\Theta) = e^{2\Theta} - 1,$$

$$\delta\mathcal{J}[F] = 0 \quad \text{with} \quad F(0) = F'(0) = F''(0) = 0 \quad \Longrightarrow \quad F = \Pi_{\geq 3}\mathcal{R}.$$

Consequently,

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right)$$

is not merely a Taylor-adjusted ansatz. It is the minimal null-admissible curvature functional selected by constrained variational projection from the raw temporal-efficiency ratio.

F Appendix F: Toward Many-Body Scalar-Time Coherence Dynamics

The present work derives the globally constrained scalar-time potential and its leading asymptotic spectral consequences for localized bound coherence sectors. The resulting effective spectral structure naturally produces shell hierarchy, subshell-sensitive coherence deformation, and thresholded shell reorganization behavior.

However, the current treatment remains fundamentally a leading-order asymptotic single-sector reduction. The present framework therefore does not yet constitute a complete many-body atomic theory.

In particular, several physically important effects remain outside the scope of the current derivation:

1. electron-electron screening,
2. exchange interactions,
3. relativistic fine structure,
4. spin-orbit coupling,
5. molecular bonding,
6. collective many-body coherence effects,
7. nuclear-electronic coupling structure.

The purpose of the present appendix is to clarify why these omissions do not contradict the central structural claims of the paper, while also outlining how such effects may emerge naturally within the scalar-time coherence framework.

The globally closed scalar-time potential derived in the main text determines the leading curvature hierarchy:

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

These curvature contributions generate the asymptotic inverse-radial structure

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}),$$

which in turn produces the leading shell-generating and coherence-binding sectors.

Within the present treatment, each localized coherence sector is analyzed independently through asymptotic fluctuation structure about a static scalar-time background

$$\Theta_0(r).$$

A full many-body theory would instead require coupled scalar-time coherence sectors, where the realized scalar-time configuration depends self-consistently upon the entire coherence distribution:

$$\Theta(x^\mu) = \Theta_{\text{nucleus}} + \sum_i \Theta_i + \Theta_{\text{collective}}.$$

In such a framework, screening effects would arise because neighboring coherence sectors modify the effective scalar-time curvature experienced by a given bound sector. The effective coefficients

$$\kappa \qquad \text{and} \qquad \beta$$

would therefore become environment-dependent collective quantities rather than isolated single-sector parameters.

Similarly, exchange interactions would not necessarily require separate phenomenological postulates. Since TSFT treats bound structure as coherence organization within a shared scalar-time field, exchange structure may emerge through admissibility constraints on overlapping coherence sectors and their associated spectral closure conditions.

Relativistic fine structure and spin-dependent effects would likewise require extension of the present asymptotic reduction beyond the leading nonrelativistic fluctuation operator. In particular, future work will need to incorporate:

1. first-order factorization structure,
2. SU(2)-covariant coherence sectors,
3. relativistic scalar-time closure corrections,
4. spin-dependent curvature coupling.

These structures were developed independently in earlier TSFT spectral-geometry work, but have not yet been unified with the globally closed scalar-time potential derived in the present paper.

Molecular structure introduces an additional level of collective organization. Within the scalar-time framework, molecular bonding would correspond not merely to electrostatic interaction, but to partially shared coherence-allocation structure across neighboring bound sectors. Stable molecular organization would therefore represent collective minima of the global scalar-time curvature functional across multi-center coherence backgrounds.

The present work should therefore be interpreted as establishing the leading global closure geometry underlying scalar-time coherence organization rather than as a completed atomic or chemical theory.

Its principal contribution is more foundational: the scalar-time potential is no longer introduced phenomenologically, but is constrained by temporal-efficiency geometry itself. The asymptotic spectral structures governing shell hierarchy and subshell deformation then emerge from this globally constrained curvature structure.

Future work will determine whether the same closure geometry can successfully generate many-body screening behavior, exchange structure, relativistic corrections, molecular organization, and higher-order chemical periodicity within a unified scalar-time coherence framework.

G Appendix G: Global Closure and Configuration-Level Realization

The main text derives a globally closed scalar-time potential,

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right),$$

with fixed null-sector curvature hierarchy

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*.$$

At first sight, this may appear to conflict with earlier atomic-sector formulations in which the subleading coherence parameter was written as configuration dependent:

$$\beta = B\eta_{\text{config}}^2.$$

The purpose of this appendix is to show that these two statements are not contradictory. They refer to different levels of structure.

The global potential determines the admissible scalar-time curvature law. A localized bound configuration then realizes a particular scalar-time background,

$$\Theta_0(r),$$

within that global law. The asymptotic form of such a background is

$$\Theta_0(r) = -\frac{A_{\text{config}}}{r} + O(r^{-2}), \qquad A_{\text{config}} > 0.$$

The coefficient

$$A_{\text{config}}$$

is not a new potential parameter. It is the amplitude of the realized coherent background. It depends on the selected bound configuration, boundary conditions, and total temporal allocation of the system.

Expanding the globally closed potential about the null sector gives

$$V''(\Theta) = 4V_* \left(e^{2\Theta} - 1\right).$$

For the realized background,

$$\Theta_0(r) = -\frac{A_{\text{config}}}{r} + O(r^{-2}),$$

we obtain

$$V''(\Theta_0(r)) = -\frac{8A_{\text{config}}V_*}{r} + \frac{8A_{\text{config}}^2 V_*}{r^2} + O(r^{-3}).$$

Therefore,

$$\kappa_{\rm config} = 8A_{\rm config}V_*,$$

and

$$\beta_{\rm config} = 8A_{\rm config}^2 V_*.$$

Thus the global closure fixes the functional form and curvature hierarchy, while the realized background amplitude determines the effective spectral residue observed within a particular coherence sector.

This resolves the apparent dichotomy:

$$\boxed{\text{Global closure fixes } V(\Theta), \qquad \text{configuration realization fixes } A_{\rm config}.}$$

Consequently,

$$\boxed{\beta_{\rm config} = 8A_{\rm config}^2 V_*}$$

is configuration dependent even though

$$V(\Theta)$$

is globally fixed.

This is directly analogous to the distinction between an operator and a realized state. The operator defines the admissible spectral structure, while the state determines which sector of that structure is occupied. In TSFT language, the globally closed potential defines the admissible coherence landscape, while a realized scalar-time background selects a stable coherence branch within that landscape.

The earlier atomic-sector notation

$$\beta = B\eta_{\rm config}^2$$

may therefore be reinterpreted as an effective configuration-level parametrization of the realized background amplitude. Comparing

$$\beta_{\rm config} = 8A_{\rm config}^2 V_*$$

with

$$\beta = B\eta_{\rm config}^2,$$

we identify

$$B\eta_{\rm config}^2 = 8A_{\rm config}^2 V_*.$$

Equivalently,

$$A_{\rm config}^2 = \frac{B}{8V_*}\eta_{\rm config}^2.$$

Thus

$$A_{\rm config}$$

is the background-amplitude realization of configuration-level temporal allocation.

This interpretation preserves both results:

$$V^{(4)}(0) = 16V_*$$

is globally fixed by the null-normalized scalar-time potential, while

$$\beta_{\text{config}}$$

varies because different coherent configurations realize different asymptotic amplitudes

$$A_{\text{config}}.$$

Therefore, the prior atomic-periodicity framework and the present global-closure framework are consistent when interpreted as different layers of the same structure:

$$\text{global law} \longrightarrow \text{admissible curvature hierarchy} \longrightarrow \text{configuration-level background} \longrightarrow \text{re}$$

The global potential is not reselected for each atom or bound system. Rather, each bound system realizes a different stable scalar-time background within the same global potential.

In this sense, the scalar-time field behaves like a coherence-selection system. The global closure defines the possible spectral geometry, while realized bound configurations select stable branches of that geometry through their temporal-allocation structure.

This resolves the apparent tension between global closure and configuration-dependent atomic ordering. The former belongs to the law-level scalar-time potential; the latter belongs to the realized coherence sector.

H Appendix H: Numerical Analysis and Sourced Background Interpretation

The globally closed scalar-time potential derived in the present work is

$$V(\Theta) = V_* \left(e^{2\Theta} - 1 - 2\Theta - 2\Theta^2\right).$$

The purpose of this appendix is to summarize several direct numerical consequences of this closed form and to clarify the physical interpretation of the asymptotic background used in the atomic-sector reduction.

H.1 H.1 Numerical Verification of the Null-Sector Expansion

Expanding the potential about the null sector gives

$$V(\Theta) = V_* \left(\frac{4}{3}\Theta^3 + \frac{2}{3}\Theta^4 + O(\Theta^5) \right).$$

Direct numerical evaluation confirms rapid convergence of the expansion near

$$\Theta = 0.$$

For

$$V_* = 1,$$

one obtains:

$$\Theta = 0.01 \quad \Longrightarrow \quad V(\Theta) \approx 1.3334 \times 10^{-6},$$

$$\Theta = 0.1 \quad \Longrightarrow \quad V(\Theta) \approx 1.3400 \times 10^{-3},$$

$$\Theta = 0.5 \quad \Longrightarrow \quad V(\Theta) \approx 1.802 \times 10^{-1}.$$

The cubic-quartic expansion agrees with the exact closed form to numerical precision in the null-sector regime.

Differentiation of the closed potential gives

$$V^{(3)}(0) = 8V_*, \qquad V^{(4)}(0) = 16V_*,$$

confirming the curvature hierarchy used throughout the main text.

The first derivative is

$$V'(\Theta) = 2V_* \left(e^{2\Theta} - 1 - 2\Theta \right).$$

Since

$$e^{2\Theta} \geq 1 + 2\Theta,$$

with equality only at

$$\Theta = 0,$$

it follows that

$$V'(\Theta) \geq 0,$$

with the null sector as the unique stationary minimum.

H.2 H.2 Numerical Structure of the Static Background Equation

The static scalar-time background equation is

$$\Theta_0''(r) + \frac{2}{r}\Theta_0'(r) = V'(\Theta_0(r)).$$

The main text uses the asymptotic form

$$\Theta_0(r) \sim -\frac{A}{r}, \qquad r \to \infty,$$

to derive the effective fluctuation operator governing the atomic spectral sector.

To test whether globally regular nonlinear solutions exist for the closed potential, the full nonlinear equation was integrated numerically for finite central amplitudes

$$\Theta_0(0) = \Theta_c, \qquad \Theta_0'(0) = 0.$$

For representative choices

$$\Theta_c = \pm 0.1, \pm 0.5, \pm 1, \pm 2,$$

all nontrivial solutions either:

1. decay rapidly back toward the vacuum sector, or
2. diverge once the field enters positive-Θ regions where

   $$V'(\Theta) > 0$$

   grows exponentially.

No smooth globally regular nontrivial localized solution approaching

$$\Theta = 0$$

at spatial infinity was found.

This behavior is consistent with the elliptic maximum principle for equations of the form

$$\nabla^2\Theta = V'(\Theta), \qquad V'(\Theta) \geq 0.$$

Consequently, the asymptotic background

$$\Theta_0(r) \sim -\frac{A}{r}$$

used in the atomic-sector analysis should not be interpreted as a vacuum soliton-like configuration. Rather, it behaves as a sourced coherence background analogous to the

Coulomb solution of Poisson theory.

H.3 H.3 Sourced Coherence Interpretation

The numerical analysis therefore suggests that stable bound scalar-time configurations require an effective coherence source term.

The static field equation should more generally be interpreted as

$$\nabla^2\Theta = V'(\Theta) + \rho_\Theta,$$

where

$$\rho_\Theta$$

represents an effective scalar-time coherence source associated with localized bound structure.

Within this interpretation:

$$\Theta = 0$$

represents the unique source-free maximum-efficiency vacuum sector, while localized matter corresponds to sourced internally retained temporal-allocation sectors.

The asymptotic form

$$\Theta_0(r) \sim -\frac{A}{r}$$

then plays the role of a sourced coherence background rather than a free nonlinear vacuum excitation.

This interpretation naturally explains why the asymptotic reduction used in the main text successfully generates Coulomb-like shell structure:

$$V''(\Theta_0(r)) = -\frac{\kappa}{r} + \frac{\beta}{r^2} + O(r^{-3}),$$

with

$$\kappa = 8AV_*, \qquad \beta = 8A^2V_*.$$

The shell-generating and subshell-generating sectors therefore emerge from the same globally closed curvature hierarchy, while the realized asymptotic background amplitude

$$A$$

is determined by the sourced coherence configuration.

H.4 H.4 Numerical Spectral Illustration

For illustrative parameters

$$V_* = 1, \qquad A = \frac{1}{2},$$

the effective asymptotic coefficients become

$$\kappa = 4, \qquad \beta = 2.$$

The leading spectral structure is then

$$\epsilon_n^{(0)} = -\frac{\kappa^2}{4n^2},$$

with the perturbative coherence-binding correction

$$\Delta\epsilon_{n\ell} = -\frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}.$$

Representative levels are:

$$\epsilon_{2s} \approx -1.031, \qquad \epsilon_{2p} \approx -1.015,$$

$$\epsilon_{3s} \approx -0.462, \qquad \epsilon_{3d} \approx -0.450.$$

Low-ℓ states are therefore preferentially stabilized relative to high-ℓ states, exactly as predicted by the asymptotic scalar-time coherence deformation.

Thresholded shell inversion occurs when

$$\beta \gtrsim 2.958,$$

equivalently,

$$A^2 V_* \gtrsim \frac{105}{284}.$$

Thus the globally closed scalar-time potential naturally generates shell hierarchy, subshell deformation, and thresholded spectral reorganization from a single curvature structure without imposing empirical shell-ordering rules externally.

H.5 H.5 Interpretation

The numerical analysis supports the following interpretation of the scalar-time framework:

1. The null sector

   $$\Theta = 0$$

   is the unique source-free maximum-efficiency vacuum.

2. The globally closed scalar-time potential fixes the admissible curvature hierarchy.

3. Localized matter corresponds to sourced internally retained temporal-allocation backgrounds.

4. Shell structure and subshell organization emerge asymptotically from the sourced coherence geometry induced by the globally closed potential.

The present work therefore does not yet constitute a full nonlinear many-body atomic theory. Rather, it establishes the globally constrained scalar-time curvature structure from which the leading asymptotic spectral organization naturally emerges.