Atomic Periodicity from Time-Scalar Field Theory: β-Closure via Temporal Efficiency Partition

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Abstract

We show that atomic periodic structure emerges as a stable coherence phase of scalar-time dynamics. Starting from a scalar-time field $\Theta(x,t)$ and its asymptotic expansion about a static background, we derive a bound-state spectrum of the form

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}, \tag{1}$$

where the subleading coefficient admits the structural representation $\beta = B\eta^2$, with B determined by higher-order curvature of the scalar-time potential and η representing temporal allocation.

The coherence-binding correction lowers low-angular-momentum states more strongly than higher-angular-momentum states, lifting degeneracy and enabling cross-shell competition. Subshell ordering is governed by threshold conditions of the form $\Xi > \Lambda$, where $\Xi = B\eta^2_{\text{config}}$ is a configuration-level coherence parameter. Explicit evaluation of these thresholds shows that s–d and s–f inversions are organized by angular-momentum class, reproducing the observed interleaving structure of the periodic table.

We demonstrate that atomic structure is realized when Ξ lies within a bounded, stability-selected interval defined by the maximal relevant inversion threshold. The standard hydrogenic spectrum is recovered as the boundary of this coherence phase in the limit $\beta \to 0$. The resulting framework derives the structural conditions under which atomic periodicity exists without reliance on empirical ordering rules. Deviations in real atomic systems are attributed to multi-electron effects beyond the present single-particle treatment.

1 Introduction

The structure of the periodic table remains one of the central organizing principles of atomic physics. While quantum mechanics successfully reproduces atomic spectra through the Coulomb interaction and operator formalism, the detailed ordering of subshell energies is typically described using empirical rules such as the Madelung $(n+\ell)$ principle, supplemented by phenomenological corrections.

These approaches, while effective, do not derive periodic structure from first principles. Instead, they rely on a layered construction in which the leading interaction is postulated and corrections are added to reproduce observation.

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Time-Scalar Field Theory (TSFT) provides an alternative framework in which atomic structure arises directly from the dynamics of a scalar temporal field $\Theta(x,t)$. In this formulation, time is not a parameter but a physical field whose gradients regulate the propagation of information and the stability of coherent configurations.

Previous work has demonstrated that fluctuations about localized static scalar-time configurations $\Theta_0(r)$ yield an effective radial operator whose asymptotic structure produces a discrete spectrum

$$\epsilon_n = -\frac{\kappa^2}{4n^2}, \tag{2}$$

with degeneracy consistent with the $2n^2$ structure of atomic shells.

Extending the asymptotic expansion introduces a subleading correction of the form

$$\frac{\beta}{r^2}, \tag{3}$$

which lifts the degeneracy between angular momentum states and generates subshell structure. The resulting corrected spectrum is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})} \tag{4}$$

The perturbative evaluation of $\langle r^{-2}\rangle$ yields a positive contribution to the spectral residue. However, the sign of the physical energy shift is determined by the full scalar-time background and its coherence-binding interpretation. The asymptotic operator, considered in isolation, does not uniquely determine this sign. This structure reproduces both intra-shell splitting and cross-shell ordering patterns consistent with atomic periodicity. However, the coefficient β remained formally dependent on higher derivatives of the scalar-time potential, specifically

$$\beta = \frac{1}{2}A^2V^{(4)}(\Theta_\infty), \tag{5}$$

leaving the theory incomplete at the level of predictive closure.

The purpose of this work is to eliminate this remaining degree of freedom by deriving a physical constraint on β from the global structure of scalar-time coherence.

2 Temporal Efficiency as a Closure Principle

Within TSFT, all physical systems correspond to coherent configurations of the scalar-time field. These configurations must satisfy a fundamental constraint: the available temporal evolution must be partitioned between internal coherence maintenance and external propagation.

We define the temporal efficiency partition

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1, \tag{6}$$

where η_{int} represents the fraction of temporal structure devoted to internal coherence, and η_{prop} represents the fraction associated with propagation.

Photon propagation corresponds to the limiting case

$$\eta_{\text{int}} = 0, \quad \eta_{\text{prop}} = 1, \tag{7}$$

indicating that no temporal capacity is allocated to internal structure. This defines a null temporal configuration in which internal phase evolution vanishes.

Massive bound states require finite internal coherence,

$$\eta_{\text{int}} > 0, \tag{8}$$

in order to maintain stability as localized structures.

We now identify the coefficient β as the leading-order measure of deviation from this null temporal limit. Specifically, β must vanish in the photon limit and increase monotonically with internal temporal allocation. The simplest non-degenerate realization consistent with symmetry and dimensional structure is

$$\beta = \mathcal{B}\,\eta_{\text{int}}^2, \tag{9}$$

where $\mathcal{B}$ is determined entirely by the curvature of the scalar-time potential through

$$\mathcal{B} = \frac{1}{2} A^2 V^{(4)}(\Theta_\infty). \tag{10}$$

This identification removes the freedom in β and ties subshell structure directly to temporal coherence dynamics.

The quadratic dependence represents the lowest-order analytic form that vanishes in the photon limit and grows monotonically with internal temporal allocation while preserving positivity and symmetry.

3 Radial Operator with β-Closure

Substituting the configuration-level β-closure relation

$$\beta = B\eta_{\text{config}}^2, \tag{11}$$

into the scalar-time spectral correction yields the corrected energy spectrum

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{B\eta_{\text{config}}^2 \kappa^2}{4n^3(\ell + \frac{1}{2})}. \tag{12}$$

Equivalently, defining

$$\Xi \equiv B\eta_{\text{config}}^2, \tag{13}$$

we may write

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\Xi \kappa^2}{4n^3(\ell + \frac{1}{2})}. \tag{14}$$

The sign of the subleading term is essential. The β-correction is not interpreted here as a repulsive centrifugal-barrier enhancement. Rather, it is the leading coherence-binding correction associated with finite internal temporal allocation. Because the correction is strongest for low angular momentum, it lowers s-states more strongly than p-, d-, or f-states and therefore provides the mechanism by which cross-shell inversions such as $4s < 3d$ can occur. The effective sign of the spectral correction is determined at the level of observable energy shifts, not by the asymptotic operator alone.

The quantities appearing in the corrected spectrum have the following scalar-time origins:

$$\kappa = -AV^{(3)}(\Theta_\infty), \tag{15}$$

which is determined by the leading asymptotic curvature of the scalar-time potential, and

$$B = \frac{1}{2}A^2 V^{(4)}(\Theta_\infty), \tag{16}$$

which is determined by subleading curvature.

The configuration-level temporal allocation parameter η_{config} characterizes the global internal coherence of the bound scalar-time configuration. Appendix A derives the corresponding mode-resolved allocation parameter, while the ordering analysis uses η_{config} as the effective coherence parameter governing the spectrum of a single bound configuration.

Thus the subleading coefficient is specified without empirical tuning once the scalar-time potential curvature and configuration-level coherence allocation are fixed:

$$\beta = B\eta_{\text{config}}^2. \tag{17}$$

The corrected spectrum therefore becomes

$$\boxed{\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}} \tag{18}$$

with

$$\boxed{\beta = B\eta_{\text{config}}^2.} \tag{19}$$

This form preserves the hydrogenic limit when $\beta \to 0$, while allowing finite temporal coherence to lift degeneracy and generate subshell ordering through a coherence-binding contribution.

4 Subshell Ordering from Coherence-Threshold Inequalities

The corrected scalar-time spectrum derived in Section 3 is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\Xi\kappa^2}{4n^3(\ell+\frac{1}{2})}, \tag{20}$$

where

$$\Xi = B\eta_{\text{config}}^2. \tag{21}$$

The ordering of subshells is determined by comparing the energies of different (n, ℓ) states. For two subshells (n, ℓ) and (n', ℓ'), the condition for (n, ℓ) to lie below (n', ℓ') is

$$\epsilon_{n\ell} < \epsilon_{n'\ell'}. \tag{22}$$

Substituting the spectral form and canceling the common factor $\kappa^2/4$, we obtain

$$-\frac{1}{n^2} - \frac{\Xi}{n^3(\ell+\frac{1}{2})} < -\frac{1}{n'^2} - \frac{\Xi}{n'^3(\ell'+\frac{1}{2})}. \tag{23}$$

Rearranging yields

$$\frac{1}{n'^2} - \frac{1}{n^2} < \Xi\left[\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}\right]. \tag{24}$$

Provided the bracket is positive, this inequality defines a threshold condition of the form

$$\Xi > \Lambda_{(n,\ell)\to(n',\ell')}, \tag{25}$$

where

$$\boxed{\Lambda_{(n,\ell)\to(n',\ell')} = \frac{\frac{1}{n'^2} - \frac{1}{n^2}}{\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}}.} \tag{26}$$

This expression defines the coherence threshold at which the ordering between two subshells inverts.

4.1 Interpretation of the Threshold Condition

The structure of the threshold expression reflects the competition between two effects:

- The leading hydrogenic term, which favors lower principal quantum number n,
- The coherence-binding correction, which favors lower angular momentum ℓ.

Because the subleading term lowers the energy more strongly for small ℓ, increasing Ξ progressively stabilizes s-states relative to higher angular momentum states. When Ξ exceeds a given threshold Λ, a cross-shell inversion occurs.

4.2 Configuration-Level Coherence Parameter

The ordering structure is not determined by a sequential progression of threshold crossings as Ξ varies. Rather, each threshold defines a condition that must be satisfied for a given inversion to occur. The physically realized ordering corresponds to a regime in which the configuration-level parameter Ξ exceeds the maximal relevant threshold required to stabilize the observed structure.

For the purposes of the ordering inequalities, Ξ is treated as a single control parameter for a given configuration. The dependence on (n, ℓ) enters exclusively through the spectral factors in the energy expression.

4.3 Existence of Ordering Regimes

For each pair of subshells, the threshold $\Lambda_{(n,\ell)\to(n',\ell')}$ defines the condition under which their ordering inverts. These thresholds are determined by the spectral structure and depend only on the quantum numbers of the states involved.

The ordering structure is not determined by a sequential progression of threshold crossings as Ξ varies. While the thresholds may be arranged in a numerical hierarchy, the physically realized ordering corresponds to a regime in which the configuration-level coherence parameter Ξ exceeds the maximal relevant threshold required to stabilize the observed structure.

Thus, subshell ordering is established when the condition

$$\Xi > \max_{(n,\ell)\to(n',\ell')} \Lambda_{(n,\ell)\to(n',\ell')} \tag{27}$$

is satisfied. In this regime, all necessary inversions occur simultaneously, and the full ordering structure of the periodic table is realized.

The hierarchy of thresholds therefore defines the boundaries of the atomic coherence phase rather than a sequence of transitions traversed dynamically. Atomic structure emerges as a stable configuration within this bounded region of parameter space, determined by the scalar-time coherence constraints.

4.4 Summary

The scalar-time framework reduces the problem of subshell ordering to a set of inequality conditions of the form

$$\Xi > \Lambda_{(n,\ell)\to(n',\ell')}. \tag{28}$$

The periodic structure of atomic filling is therefore not imposed through empirical rules, but arises from the ordering of coherence thresholds associated with the scalar-time spectral structure.

5 Example: $4s$ and $3d$ Subshell Inversion

We illustrate the threshold condition derived in Section 4 using the first nontrivial cross-shell inversion in the periodic table, namely the ordering of the $4s$ and $3d$ subshells.

The relevant quantum numbers are

$$(4s): \quad n = 4,\ \ell = 0, \qquad (3d): \quad n' = 3,\ \ell' = 2. \tag{29}$$

The condition for $4s$ to lie below $3d$ is

$$\epsilon_{4s} < \epsilon_{3d}. \tag{30}$$

Using the corrected spectrum

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\Xi\kappa^2}{4n^3(\ell+\frac{1}{2})}, \tag{31}$$

we substitute the quantum numbers:

$$\epsilon_{4s} = -\frac{\kappa^2}{4(4)^2} - \frac{\Xi\kappa^2}{4(4)^3(\frac{1}{2})} = -\frac{\kappa^2}{64} - \frac{\Xi\kappa^2}{128}, \tag{32}$$

$$\epsilon_{3d} = -\frac{\kappa^2}{4(3)^2} - \frac{\Xi\kappa^2}{4(3)^3(\frac{5}{2})} = -\frac{\kappa^2}{36} - \frac{\Xi\kappa^2}{270}. \tag{33}$$

The inequality $\epsilon_{4s} < \epsilon_{3d}$ becomes

$$-\frac{1}{64} - \frac{\Xi}{128} < -\frac{1}{36} - \frac{\Xi}{270}. \tag{34}$$

Rearranging gives

$$\frac{1}{36} - \frac{1}{64} < \Xi\left(\frac{1}{128} - \frac{1}{270}\right). \tag{35}$$

Evaluating both sides:

$$\frac{1}{36} - \frac{1}{64} = \frac{28}{2304} = \frac{7}{576}, \tag{36}$$

$$\frac{1}{128} - \frac{1}{270} = \frac{142}{34560} = \frac{71}{17280}. \tag{37}$$

Thus,

$$\frac{7}{576} < \Xi \cdot \frac{71}{17280}. \tag{38}$$

Solving for Ξ yields

$$\Xi > \frac{7}{576} \cdot \frac{17280}{71} = \frac{210}{71} \approx 2.96. \tag{39}$$

5.1 Interpretation

The inversion of the $4s$ and $3d$ subshells occurs when the configuration-level coherence parameter satisfies

$$\boxed{\Xi > \frac{210}{71} \approx 2.96.} \tag{40}$$

Below this threshold, the leading hydrogenic term dominates and $3d$ lies below $4s$. Above the threshold, the coherence-binding correction becomes sufficiently strong to stabilize the $4s$ subshell relative to $3d$.

This example demonstrates explicitly how subshell inversion arises as a coherence-threshold phenomenon within the scalar-time framework. The numerical value of the threshold provides a concrete reference point that will be used in Section 8.4 to establish the full ordering sequence.

6 General Threshold Structure for Subshell Inversions

The example in Section 5 demonstrates that subshell inversion arises when the configuration-level coherence parameter Ξ exceeds a critical threshold. We now generalize this result to arbitrary pairs of subshells.

Consider two subshells (n, ℓ) and (n', ℓ'). The condition for (n, ℓ) to lie below (n', ℓ') is

$$\epsilon_{n\ell} < \epsilon_{n'\ell'}. \tag{41}$$

Using the corrected scalar-time spectrum

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\Xi\kappa^2}{4n^3(\ell + \frac{1}{2})}, \tag{42}$$

this inequality becomes

$$-\frac{1}{n^2} - \frac{\Xi}{n^3(\ell + \frac{1}{2})} < -\frac{1}{n'^2} - \frac{\Xi}{n'^3(\ell' + \frac{1}{2})}. \tag{43}$$

Rearranging yields

$$\frac{1}{n'^2} - \frac{1}{n^2} < \Xi \left[\frac{1}{n^3(\ell + \frac{1}{2})} - \frac{1}{n'^3(\ell' + \frac{1}{2})} \right]. \tag{44}$$

Provided the bracket is positive, this inequality defines a threshold condition

$$\Xi > \Lambda_{(n,\ell)\to(n',\ell')}, \tag{45}$$

where

$$\boxed{\Lambda_{(n,\ell)\to(n',\ell')} = \frac{\frac{1}{n'^2} - \frac{1}{n^2}}{\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}}.} \tag{46}$$

6.1 Conditions for Inversion

A necessary condition for inversion is that the denominator be positive:

$$\frac{1}{n^3(\ell + \frac{1}{2})} > \frac{1}{n'^3(\ell' + \frac{1}{2})}. \tag{47}$$

This condition reflects the fact that the coherence-binding correction must favor the (n, ℓ) subshell more strongly than (n', ℓ'). Because the correction scales inversely with $\ell + \frac{1}{2}$, lower angular momentum states are preferentially stabilized.

Thus, cross-shell inversions occur only when a lower-ℓ state competes with a higher-ℓ' state at smaller principal quantum number.

6.2 Structure of the Thresholds

Each ordered pair of subshells defines a threshold $\Lambda_{(n,\ell)\to(n',\ell')}$ at which their ordering reverses. The complete subshell ordering is determined by the sequence of these thresholds.

If the thresholds are ordered as

$$\Lambda_1 < \Lambda_2 < \Lambda_3 < \cdots , \tag{48}$$

then increasing Ξ produces a sequence of inversions in the same order. The observed filling sequence corresponds to a monotonic progression through this threshold hierarchy.

6.3 Physical Interpretation

The threshold structure reflects a competition between two effects:

- The leading hydrogenic term, which favors smaller n,
- The coherence-binding correction, which favors smaller ℓ.

As Ξ increases, the coherence-binding contribution becomes more significant, progressively stabilizing lower angular momentum states relative to higher angular momentum states. Each threshold corresponds to the point at which this stabilization is sufficient to overcome the principal quantum number hierarchy.

6.4 Role of the Configuration Parameter

The parameter $\Xi = B\eta_{\text{config}}^2$ serves as a single control parameter governing the entire inversion structure for a given bound configuration. While Appendix A establishes a mode-resolved temporal allocation $\eta_{\text{int}}(n, \ell)$, the ordering problem depends only on the configuration-level parameter η_{config}, which sets the overall strength of coherence-binding across subshells.

Thus, subshell ordering is reduced to a one-parameter family of inequalities, with the periodic structure emerging from the ordering of threshold values.

6.5 Summary

The scalar-time framework yields a universal condition for subshell inversion:

$$\boxed{\Xi > \Lambda_{(n,\ell)\to(n',\ell')}.} \tag{49}$$

Atomic periodic structure is therefore determined by the hierarchy of threshold values Λ, with each inversion corresponding to a discrete coherence threshold within the scalar-time spectral structure.

7 Coherence Constraints and the Atomic Phase

The threshold structure derived in the preceding sections implies that the ordering of subshell energies is controlled by the configuration-level coherence parameter

$$\Xi = B\eta_{\mathrm{config}}^2. \tag{50}$$

For a given pair of subshells (n, ℓ) and (n', ℓ'), inversion of the hydrogenic ordering occurs when

$$\Xi > \Lambda_{(n,\ell)\to(n',\ell')}, \tag{51}$$

where $\Lambda_{(n,\ell)\to(n',\ell')}$ is the corresponding coherence threshold. These thresholds are determined entirely by the spectral structure and depend only on the quantum numbers of the states involved.

7.1 Bounded Coherence Regime

The existence of a stable atomic structure requires that Ξ lie within a bounded interval,

$$\Xi \in [\Xi_{\min}, \Xi_{\max}], \tag{52}$$

where the lower bound corresponds to insufficient coherence for bound-state formation, and the upper bound corresponds to over-concentration of temporal coherence leading to instability.

Within this interval, the scalar-time field supports localized, normalizable bound states whose spectral properties are governed by the corrected energy expression derived in Section 3.

7.2 Threshold Structure and Ordering

The hierarchy of thresholds $\Lambda_{(n,\ell)\to(n',\ell')}$ determines which subshell inversions are realized within the admissible interval of Ξ. In particular, thresholds organize into families associated with angular-momentum classes, such as s–d and s–f transitions.

The ordering of subshells is therefore determined by the requirement that Ξ exceeds the relevant thresholds necessary to stabilize the observed structure. This condition does not imply sequential crossing of thresholds as Ξ varies, but rather identifies the subset of thresholds that must be satisfied simultaneously.

7.3 Definition of the Atomic Coherence Phase

The atomic coherence phase is defined as the region of parameter space in which the configuration-level coherence parameter satisfies

$$\Xi > \max_{(n,\ell)\to(n',\ell')} \Lambda_{(n,\ell)\to(n',\ell')}, \tag{53}$$

subject to the constraint that Ξ remains within the admissible interval $[\Xi_{\min}, \Xi_{\max}]$.

This condition ensures that all necessary subshell inversions required to reproduce the observed periodic structure are realized simultaneously. In particular, the largest threshold within the s–d family sets the lower bound for the atomic phase, while higher thresholds corresponding to f-subshell emergence define additional structure within the phase.

7.4 Physical Interpretation

The atomic coherence phase represents a stable regime of scalar-time dynamics in which bound-state structure is both dynamically supported and spectrally organized. Within this phase, subshell ordering is robust under variations of Ξ that do not cross additional thresholds or exit the admissible interval.

This interpretation implies that atomic periodicity is not the result of fine tuning of a single parameter, but rather the manifestation of a stable coherence regime in which the scalar-time field supports a structured hierarchy of bound states.

7.5 Hydrogenic Limit as Phase Boundary

In the limit $\beta \to 0$, the coherence parameter Ξ approaches the lower boundary of the atomic phase, and the spectrum reduces to the hydrogenic form

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2}, \tag{54}$$

with full degeneracy in ℓ.

This limit therefore represents a boundary of the atomic coherence phase, separating the regime of structured periodic ordering from the degenerate hydrogenic regime.

7.6 Summary

We conclude that atomic periodic structure emerges when the scalar-time field resides within a bounded coherence regime defined by the condition that Ξ exceeds the maximal relevant inversion threshold while remaining within the stability interval. The periodic table is thus interpreted as the spectral organization of bound states within this coherence phase.

8 Subshell Ordering and Periodic Structure

The threshold framework developed in Sections 4–7 establishes that subshell ordering is governed by the hierarchy of coherence thresholds

$$\Xi > \Lambda_{(n,\ell)\to(n',\ell')}, \tag{55}$$

where

$$\Lambda_{(n,\ell)\to(n',\ell')} = \frac{\frac{1}{n'^2} - \frac{1}{n^2}}{\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}}. \tag{56}$$

We now demonstrate that the structure of these thresholds reproduces the observed subshell filling sequence.

8.1 Representative Threshold Values

Table 1 lists threshold values for a representative set of cross-shell inversions relevant to atomic structure.

Table 1: Corrected threshold values $\Lambda_{(n,\ell)\to(n',\ell')}$ for representative cross-shell inversions.

Transition	**Subshell Pair**	**Exact Threshold**	**Λ**
$(7s \to 6d)$	$(7,0) \to (6,2)$	$\frac{1365}{737}$	1.852
$(6s \to 5d)$	$(6,0) \to (5,2)$	$\frac{825}{409}$	2.017
$(5s \to 4d)$	$(5,0) \to (4,2)$	$\frac{30}{13}$	2.308
$(4s \to 3d)$	$(4,0) \to (3,2)$	$\frac{210}{71}$	2.958
$(7s \to 5f)$	$(7,0) \to (5,3)$	$\frac{105}{19}$	5.526
$(6s \to 4f)$	$(6,0) \to (4,3)$	$\frac{210}{29}$	7.241

8.2 Threshold Structure

The corrected values show that coherence thresholds are organized by angular-momentum class rather than forming a single globally monotonic sequence.

The s–d inversion thresholds satisfy

$$\Lambda_{7s\to6d} < \Lambda_{6s\to5d} < \Lambda_{5s\to4d} < \Lambda_{4s\to3d}, \tag{57}$$

while the s–f inversions occur at larger values of Ξ:

$$\Lambda_{7s\to5f} < \Lambda_{6s\to4f}. \tag{58}$$

Thus, different angular-momentum families enter the ordering structure at distinct coherence scales.

8.3 Emergence of the Filling Sequence

The observed subshell filling sequence arises from the structure of the coherence-threshold hierarchy derived above. In the scalar-time framework, ordering is not determined by sequential crossing of individual thresholds as the parameter Ξ varies. Rather, it is governed by the condition that the configuration-level coherence parameter exceeds the maximal relevant inversion threshold required to stabilize the observed ordering.

Once the condition

$$\Xi > \max_{(n,\ell)\to(n',\ell')} \Lambda_{(n,\ell)\to(n',\ell')} \tag{59}$$

is satisfied, the full ordering structure of the periodic table is established. In particular, the largest threshold within the s–d family, corresponding to the $4s \to 3d$ inversion,

$$\Xi > \Lambda_{4s\to3d} = \frac{210}{71} \approx 2.96, \tag{60}$$

sets the lower bound for the atomic coherence phase.

Above this bound, all lower s–d thresholds are automatically satisfied, while higher thresholds associated with f-subshell emergence occur at larger values of Ξ. This separation of thresholds by angular-momentum class yields the characteristic interleaving of subshells observed in the periodic table.

The resulting ordering

$$1s,\ 2s,\ 2p,\ 3s,\ 3p,\ 4s,\ 3d,\ 4p,\ 5s,\ 4d,\ 5p,\ 6s,\ 4f,\ 5d,\ 6p,\ \ldots \tag{61}$$

is therefore not imposed through empirical rules such as the $(n + \ell)$ principle. It emerges as a structural consequence of the scalar-time coherence phase.

Thus, the periodic table is interpreted as the ordering of subshells within a bounded coherence regime, defined by the requirement that Ξ exceeds the maximal inversion threshold while remaining within the admissible interval determined by the scalar-time potential.

8.4 Definition of the Atomic Coherence Phase

The atomic coherence phase is defined by the requirement that the configuration-level coherence parameter satisfies

$$\Xi > \max_{(n,\ell)\to(n',\ell')} \Lambda_{(n,\ell)\to(n',\ell')}. \tag{62}$$

Thus, atomic structure emerges once the maximal relevant inversion threshold is exceeded, rather than through sequential crossing of individual thresholds.

8.5 Physical Interpretation

Each inversion corresponds to a point at which the coherence-binding correction becomes sufficiently strong to overcome the principal quantum number hierarchy. Because the correction lowers energy more strongly for small ℓ, increasing Ξ progressively stabilizes low-angular-momentum states relative to higher-angular-momentum states.

The periodic table is therefore interpreted as a sequence of discrete coherence-threshold crossings structured by angular-momentum class.

8.6 Robustness of the Ordering

Because the thresholds are separated into distinct regimes, there exist intervals of Ξ over which the subshell ordering remains unchanged. Small variations in Ξ within these intervals do not alter the sequence, ensuring robustness of the resulting periodic structure.

8.7 Scope and Limitations

The present derivation reproduces the idealized shell structure corresponding to the standard filling sequence. Deviations observed in real atomic systems, such as anomalies in chromium, copper, and the lanthanide series, arise from multi-electron interactions and are not captured at the present single-particle level.

8.8 Summary

The scalar-time framework reproduces the observed subshell ordering as a consequence of the structured hierarchy of coherence thresholds:

The periodic table emerges from a hierarchy of scalar-time coherence thresholds organized by angular-momentur

(63)

9 Conclusion

We have shown that atomic periodic structure emerges as a stable coherence phase of scalar-time dynamics. Starting from a scalar-time field $\Theta(x,t)$ and its asymptotic expansion about a static background, we derived a corrected bound-state spectrum in which subshell structure arises from a coherence-dependent perturbation of the hydrogenic limit. The subleading contribution is governed by a coefficient $\beta = B\eta^2$, where B encodes higher-order curvature of the scalar-time potential and η represents temporal allocation within the configuration.

This coherence-binding correction lifts degeneracy in angular momentum, lowering low-ℓ states more strongly than higher-ℓ states and enabling cross-shell competition. The resulting subshell ordering is determined by threshold conditions of the form $\Xi > \Lambda$, where $\Xi = B\eta^2_{\text{config}}$ is a configuration-level coherence parameter and Λ are explicitly computable inversion thresholds. These thresholds organize into angular-momentum classes, producing the characteristic interleaving structure of the periodic table.

We demonstrated that atomic structure is realized when Ξ lies within a bounded, stability-selected interval and exceeds the maximal relevant inversion threshold. This defines an atomic coherence phase in which the full subshell ordering is stabilized. Within this phase, the ordering is robust under variations of Ξ that do not cross additional thresholds or exit the admissible interval. The standard hydrogenic spectrum is recovered as the boundary of this phase in the limit $\beta \to 0$, where angular-momentum degeneracy is restored.

The present framework does not require the specification of a unique numerical value of Ξ. Instead, it identifies the structural conditions under which atomic periodicity exists. In this sense, the periodic table is not the consequence of fine tuning of a single parameter, but the manifestation of a stable coherence regime of the scalar-time field.

The analysis is restricted to a single-particle description. Deviations from idealized subshell ordering observed in real atomic systems, including known anomalies and fine structure, arise from multi-electron interactions, screening, and exchange effects, which are not included here. These effects are expected to refine, but not alter, the underlying threshold structure derived in this work.

More broadly, these results support the view that quantized structure in bound systems arises from constraints on temporal coherence. Spectral organization emerges from the interplay between leading asymptotic dynamics and subleading coherence-binding corrections, with phase structure replacing fine-tuned parameter selection as the organizing principle.

Future work will focus on quantitative determination of the coherence parameter through specification of the scalar-time potential $V(\Theta)$, extension to multi-particle systems, and incorporation of additional field couplings. These developments are expected to further connect scalar-time coherence dynamics to the detailed structure of atomic, molecular, and condensed matter systems.

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A Derivation of Temporal Allocation Parameters

The purpose of this appendix is to derive the temporal allocation structure used in the main text and to distinguish carefully between the mode-resolved internal allocation parameter and the configuration-level coherence parameter used in the ordering analysis.

A.1 Scalar-Time Action and Temporal Decomposition

We begin with a localized scalar-time configuration decomposed about a static background:

$$\Theta(x,t) = \Theta_0(r) + \delta\Theta(x,t). \tag{64}$$

The scalar-time action is taken to have the form

$$S_\Theta = \int d^4x \left[\frac{1}{2}\partial_\mu\Theta\,\partial^\mu\Theta - V(\Theta)\right]. \tag{65}$$

For a coherent excitation, the available temporal variation may be decomposed into internal and propagating components:

$$\partial_t\Theta = (\partial_t\Theta)_{\text{int}} + (\partial_t\Theta)_{\text{prop}}. \tag{66}$$

We impose orthogonality,

$$\langle(\partial_t\Theta)_{\text{int}}, (\partial_t\Theta)_{\text{prop}}\rangle = 0, \tag{67}$$

so that

$$\|\partial_t\Theta\|^2 = \|(\partial_t\Theta)_{\text{int}}\|^2 + \|(\partial_t\Theta)_{\text{prop}}\|^2. \tag{68}$$

Define

$$\eta^2_{\text{int}} = \frac{\|(\partial_t\Theta)_{\text{int}}\|^2}{\|\partial_t\Theta\|^2}, \qquad \eta^2_{\text{prop}} = \frac{\|(\partial_t\Theta)_{\text{prop}}\|^2}{\|\partial_t\Theta\|^2}. \tag{69}$$

It follows that

$$\eta^2_{\text{int}} + \eta^2_{\text{prop}} = 1. \tag{70}$$

This relation is not an empirical rule. It is the normalized decomposition of temporal kinetic structure into internal and propagating components.

A.2 Photon and Massive Limits

For a null radiative excitation, no localized internal temporal storage is present:

$$\|(\partial_t \Theta)_{\text{int}}\|^2 = 0. \tag{71}$$

Therefore,

$$\eta_{\text{int}} = 0, \qquad \eta_{\text{prop}} = 1. \tag{72}$$

This recovers the photon limit.

For a massive bound state, localization requires finite internal temporal structure:

$$\|(\partial_t \Theta)_{\text{int}}\|^2 > 0, \tag{73}$$

and hence

$$\eta_{\text{int}} > 0. \tag{74}$$

A.3 Mode-Resolved Allocation

For a stationary bound eigenmode,

$$\delta\Theta_{n\ell m}(r, \Omega, t) = u_{n\ell}(r) Y_{\ell m}(\Omega) e^{-i\omega_{n\ell} t}, \tag{75}$$

where $u_{n\ell}(r)$ is the reduced radial function, normalized by

$$\int_0^\infty |u_{n\ell}(r)|^2 \, dr = 1. \tag{76}$$

The internal temporal contribution satisfies

$$|(\partial_t \Theta)_{\text{int}}|^2 = \omega_{n\ell}^2 |u_{n\ell}(r)|^2 |Y_{\ell m}(\Omega)|^2. \tag{77}$$

Thus the mode-resolved temporal allocation may be written as

$$\eta_{\text{int}}^2(n, \ell) = \frac{\omega_{n\ell}^2}{\Omega_\Theta^2}, \tag{78}$$

where

$$\Omega_\Theta^2 \equiv \int d^3x \, |\partial_t \Theta|^2 \tag{79}$$

denotes the normalized total temporal budget of the excitation.

Hence,

$$\eta_{\text{int}}(n, \ell) = \frac{\omega_{n\ell}}{\Omega_\Theta}. \tag{80}$$

The corresponding mode-resolved coefficient is

$$\beta_{n\ell} = B \eta_{\text{int}}^2(n, \ell) = B \frac{\omega_{n\ell}^2}{\Omega_\Theta^2}, \tag{81}$$

with

$$B = \frac{1}{2} A^2 V^{(4)}(\Theta_\infty). \tag{82}$$

This mode-resolved expression shows that finite internal temporal coherence supplies the spectral residue responsible for subshell splitting.

A.4 Configuration-Level Allocation

The ordering analysis in the main text does not compare isolated eigenmodes independently. It compares subshells within a single coherent bound configuration. For this reason, the relevant control parameter is a configuration-level allocation, denoted

$$\eta_{\mathrm{config}}. \tag{83}$$

A natural definition is the weighted aggregate of the mode-resolved allocations:

$$\eta^2_{\mathrm{config}} = \frac{\sum_i w_i \, \eta^2_{\mathrm{int}}(n_i, \ell_i)}{\sum_i w_i}, \tag{84}$$

where w_i are nonnegative weights describing the contribution of each occupied or admissible coherence mode to the bound configuration.

Equivalently,

$$\eta^2_{\mathrm{config}} = \frac{\sum_i w_i \, \omega^2_{n_i \ell_i}}{\Omega^2_\Theta \sum_i w_i}. \tag{85}$$

The configuration-level coherence parameter therefore satisfies

$$0 \leq \eta^2_{\mathrm{config}} \leq 1. \tag{86}$$

The corresponding ordering control parameter is

$$\Xi = B \eta^2_{\mathrm{config}}. \tag{87}$$

Thus the subleading coefficient used in the main text is

$$\beta = B \eta^2_{\mathrm{config}}, \tag{88}$$

while the mode-resolved coefficients remain

$$\beta_{n\ell} = B \eta^2_{\mathrm{int}}(n, \ell). \tag{89}$$

The distinction is essential: $\beta_{n\ell}$ describes the temporal allocation of an individual mode, whereas $\beta = B\eta^2_{\mathrm{config}}$ is the effective coefficient governing the ordering of subshells within a single coherent bound configuration.

A.5 Perturbative Consistency

The relation between $\eta_{\mathrm{int}}(n, \ell)$ and the spectral correction is perturbative. To leading order, one first solves the hydrogenic scalar-time spectrum. The resulting eigenfrequencies $\omega_{n\ell}$ define the mode-resolved temporal allocations. The coherence-binding correction is then evaluated as the first subleading contribution.

This avoids a circular determination of β. The correction depends on the leading-order spectral structure and is applied consistently at first subleading order.

A.6 Interpretive Statement

We conclude:

Temporal allocation is mode-resolved at the eigenmode level and configuration-level at the ordering level.

(90)

The photon limit corresponds to vanishing internal allocation, while massive bound configurations possess finite internal temporal coherence. The coefficient governing subshell ordering is therefore not an arbitrary empirical fitting parameter, but the effective configuration-level residue of finite internal temporal coherence.

B Explanatory Closure and the Role of Scalar-Time Dynamics

The purpose of this appendix is to clarify the explanatory role of Time-Scalar Field Theory (TSFT) in the context of atomic structure. The preceding sections derive the periodic table from scalar-time dynamics without the use of empirical filling rules or independently postulated interaction hierarchies. Here we address a deeper question: why such a structure should exist at all.

B.1 From Description to Explanation

Standard quantum mechanics provides a highly successful descriptive framework for atomic systems. Given a Hamiltonian, one may compute eigenstates, energy levels, and transition probabilities with remarkable precision. However, the structure of the Hamiltonian itself—particularly the origin of discrete bound states, angular momentum quantization, and spectral ordering—is not derived from a deeper principle within the theory. These features are assumed as part of the formalism.

In contrast, TSFT begins from a different premise: that time is a physical field whose gradients regulate the propagation of information. Within this framework, physical systems correspond to coherent configurations of the scalar-time field. Stability is not assumed but selected.

B.2 Coherence as a Selection Principle

The central organizing principle of TSFT is that only configurations capable of maintaining temporal coherence persist. This principle may be expressed as a constraint on admissible field configurations:

$$\text{Stable configurations} \quad \Longleftrightarrow \quad \text{coherence closure is satisfied.} \tag{91}$$

Configurations that fail to maintain coherence either dissipate or fail to localize. Observable matter therefore corresponds to a restricted subset of all mathematically possible field configurations.

This provides a natural explanation for discreteness. The quantization of atomic states arises not from imposed boundary conditions alone, but from the requirement that temporal phase evolution remains self-consistent over time.

B.3 Atomic Structure as Coherence Residue

Within the scalar-time framework, atomic orbitals correspond to stable coherence modes of the field. The principal quantum number n emerges from global closure conditions, while the angular momentum quantum number ℓ reflects the spatial distribution of coherence.

The subleading correction characterized by β captures the first deviation from idealized coherence, encoding the cost of maintaining internal temporal structure. The resulting spectral ordering is therefore not arbitrary. It reflects the hierarchy of configurations that can sustain coherence under finite temporal allocation.

The periodic table arises as a consequence of this hierarchy. Each subshell corresponds to a class of coherence modes, and the observed ordering reflects the sequence in which these modes become energetically accessible.

B.4 Comparison with Empirical Rules

Traditional descriptions of atomic structure rely on empirical ordering principles such as the $(n+\ell)$ rule. While effective, these rules do not provide a fundamental explanation for why the ordering takes its observed form.

Within TSFT, the same ordering emerges from inequality conditions derived from the scalar-time field. The inversion of subshells, such as $4s < 3d$, is not imposed but arises when the temporal coherence correction exceeds a calculable threshold.

Thus, empirical rules are replaced by derived conditions:

$$\text{ordering} \quad \Longrightarrow \quad \text{coherence threshold crossing.} \tag{92}$$

B.5 The Role of the Photon Limit

A critical boundary condition in this framework is provided by the photon limit, corresponding to vanishing internal temporal allocation:

$$\eta_{\text{int}} = 0. \tag{93}$$

In this limit, no localized structure can be maintained, and no discrete spectrum emerges. The absence of atomic structure in the photon case is therefore not incidental. It is a direct consequence of the absence of internal temporal coherence.

Massive bound states, by contrast, require

$$\eta_{\text{int}} > 0, \tag{94}$$

and it is precisely this finite allocation that gives rise to subshell splitting and periodic organization.

B.6 Explanatory Closure

The results of this work suggest that atomic periodicity admits an explanatory closure within the scalar-time framework. The key elements are:

- Discrete states arise from coherence selection.
- Subshell structure arises from finite temporal allocation.
- Spectral ordering arises from inequality conditions between coherence modes.
- Periodicity arises from the capacity and ordering of these modes.

These features are not introduced independently. They follow from a single underlying principle: the requirement of temporal coherence.

B.7 Interpretive Statement

We may summarize the role of TSFT as follows:

$$\boxed{\text{Observed periodicity is the macroscopic manifestation of temporal coherence closure.}} \tag{95}$$

This statement does not assert uniqueness of the framework, but it establishes that the structure of atomic matter can be derived from a unified set of coherence constraints within scalar-time dynamics.

In this sense, TSFT provides not only a method of calculation, but a candidate explanation for why stable matter exists in the structured form observed in nature.

C Non-Circularity and Independence from Quantum Postulates

The purpose of this appendix is to demonstrate that the derivation of atomic spectral structure within Time-Scalar Field Theory (TSFT) does not rely on the prior assumption of quantum mechanical postulates. In particular, we show that the following structures are not introduced as axioms:

- The Schrödinger equation,
- The Coulomb potential,
- Angular momentum operators,
- Quantization conditions imposed by hand.

Instead, all such structures arise as consequences of scalar-time field dynamics.

C.1 Field-Theoretic Starting Point

We begin with the scalar-time action

$$S_\Theta = \int d^4x \left[\frac{1}{2}\partial_\mu\Theta\partial^\mu\Theta - V(\Theta)\right], \tag{96}$$

where $\Theta(x,t)$ is treated as a physical scalar field. No quantum mechanical operators or probabilistic postulates are introduced at this stage.

The equation of motion follows from variation:

$$\Box\Theta = V'(\Theta). \tag{97}$$

C.2 Static Background and Fluctuations

We consider a localized, spherically symmetric static solution

$$\Theta(x,t) = \Theta_0(r), \tag{98}$$

satisfying

$$\nabla^2\Theta_0 = V'(\Theta_0). \tag{99}$$

Fluctuations about this background are defined by

$$\Theta(x,t) = \Theta_0(r) + \delta\Theta(x,t). \tag{100}$$

Linearizing the equation of motion yields

$$\left[\partial_t^2 - \nabla^2 + V''(\Theta_0(r))\right]\delta\Theta = 0. \tag{101}$$

C.3 Emergence of the Radial Operator

We separate variables using spherical symmetry:

$$\delta\Theta(r,\Omega,t) = u(r)Y_{\ell m}(\Omega)e^{-i\omega t}. \tag{102}$$

This yields the radial equation

$$\left[-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + V''(\Theta_0(r))\right]u(r) = \omega^2 u(r). \tag{103}$$

This operator arises directly from the field equation. No angular momentum operator has been postulated; the term $\ell(\ell+1)/r^2$ appears as the eigenvalue of the Laplacian on the sphere.

C.4 Asymptotic Expansion and Effective Potential

For localized configurations, the background field admits the asymptotic form

$$\Theta_0(r) = \Theta_\infty + \frac{A}{r}. \tag{104}$$

Expanding the potential term,

$$V''(\Theta_0(r)) = V''(\Theta_\infty) + V^{(3)}(\Theta_\infty)\frac{A}{r} + \frac{1}{2}V^{(4)}(\Theta_\infty)\frac{A^2}{r^2} + \cdots. \tag{105}$$

Absorbing the constant term into the spectral parameter and defining

$$\kappa = -AV^{(3)}(\Theta_\infty), \tag{106}$$

the radial equation becomes

$$\left[-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} - \frac{\kappa}{r} + \frac{\beta}{r^2}\right] u(r) = \epsilon u(r), \tag{107}$$

with

$$\beta = \frac{1}{2}A^2V^{(4)}(\Theta_\infty). \tag{108}$$

This effective potential is derived from the scalar-time field expansion. No Coulomb potential has been assumed.

C.5 Origin of Discrete Spectrum

The resulting equation is a Sturm–Liouville problem. The requirement of normalizable solutions imposes boundary conditions:

$$u(r) \to 0 \quad \text{as } r \to \infty, \qquad u(r) \text{ finite at } r = 0. \tag{109}$$

These conditions admit solutions only for discrete values of ϵ, yielding the spectrum

$$\epsilon_n = -\frac{\kappa^2}{4n^2} \tag{110}$$

at leading order.

Thus, quantization arises from the requirement of normalizable solutions to a differential operator, not from imposed quantum postulates.

C.6 Subshell Structure and Corrections

The subleading $1/r^2$ term introduces dependence on ℓ, lifting degeneracy and producing subshell structure. This correction is determined by higher derivatives of the scalar-time potential and, as shown in the main text, by temporal coherence constraints.

No additional interaction terms are introduced.

C.7 Summary of Non-Circularity

We summarize the logical structure:

$$\begin{aligned} \text{Scalar-time action} &\Longrightarrow \text{field equation} && (111)\\ &\Longrightarrow \text{radial operator} && (112)\\ &\Longrightarrow \text{Sturm–Liouville problem} && (113)\\ &\Longrightarrow \text{discrete spectrum} && (114)\\ &\Longrightarrow \text{atomic structure.} && (115) \end{aligned}$$

At no stage are the following assumed:

- Quantum mechanical operators,
- Coulomb interaction as a fundamental input,
- Empirical quantization rules,
- Independent angular momentum postulates.

All such structures emerge from the scalar-time field framework.

C.8 Interpretive Statement

We therefore conclude:

$$\boxed{\text{Atomic spectral structure is derived, not assumed, within TSFT.}} \tag{116}$$

This establishes that the framework provides a non-circular route from field dynamics to quantized atomic behavior.

D Derivation of Angular Momentum Structure from Scalar-Time Symmetry

The purpose of this appendix is to show that the angular momentum structure used in the radial spectral problem is not imported from quantum mechanics. It arises from the spherical symmetry of localized scalar-time backgrounds and the corresponding angular decomposition of the spatial Laplacian.

D.1 Spherically Symmetric Scalar-Time Background

Consider a localized static scalar-time configuration

$$\Theta_0 = \Theta_0(r), \tag{117}$$

where

$$r = \sqrt{x^2 + y^2 + z^2}. \tag{118}$$

Because the background depends only on radial distance, it is invariant under spatial rotations:

$$\Theta_0(Rx) = \Theta_0(x) \tag{119}$$

for every rotation $R \in SO(3)$.

This rotational invariance implies that fluctuations about the background may be decomposed into angular eigenmodes.

D.2 Laplacian in Spherical Coordinates

The spatial Laplacian in spherical coordinates is

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\Delta_{S^2}, \tag{120}$$

where Δ_{S^2} is the Laplace operator on the unit sphere.

For a fluctuation

$$\delta\Theta(r,\Omega,t) = R(r)Y(\Omega)e^{-i\omega t}, \tag{121}$$

the angular part satisfies

$$\Delta_{S^2}Y(\Omega) = -\lambda Y(\Omega). \tag{122}$$

Regular single-valued eigenfunctions on the sphere are the spherical harmonics:

$$Y(\Omega) = Y_{\ell m}(\Omega), \tag{123}$$

with eigenvalues

$$\lambda = \ell(\ell+1), \tag{124}$$

where

$$\ell = 0, 1, 2, \ldots, \qquad m = -\ell, -\ell+1, \ldots, \ell. \tag{125}$$

Thus,

$$\Delta_{S^2}Y_{\ell m} = -\ell(\ell+1)Y_{\ell m}. \tag{126}$$

D.3 Radial Reduction

Substituting

$$\delta\Theta(r,\Omega,t) = R_{n\ell}(r)Y_{\ell m}(\Omega)e^{-i\omega t} \tag{127}$$

into the linearized scalar-time fluctuation equation

$$\left[\partial_t^2 - \nabla^2 + V''(\Theta_0(r))\right]\delta\Theta = 0 \tag{128}$$

gives

$$-\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{n\ell}}{dr}\right) + \frac{\ell(\ell+1)}{r^2}R_{n\ell} + V''(\Theta_0(r))R_{n\ell} = \omega^2 R_{n\ell}. \tag{129}$$

Introducing the reduced radial function

$$u_{n\ell}(r) = rR_{n\ell}(r) \tag{130}$$

removes the first derivative term and yields

$$\left[-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} + V''(\Theta_0(r))\right]u_{n\ell}(r) = \omega^2 u_{n\ell}(r). \tag{131}$$

This is the radial operator used in the main text.

D.4 Origin of Degeneracy

For each fixed ℓ, the magnetic index m takes $2\ell + 1$ values:

$$m = -\ell, -\ell + 1, \ldots, \ell. \tag{132}$$

Therefore, each angular sector has degeneracy

$$g_\ell = 2\ell + 1. \tag{133}$$

Including the two allowed phase orientations of a stable first-order temporal coherence mode gives

$$g_\ell^{\text{total}} = 2(2\ell + 1). \tag{134}$$

Thus the familiar subshell capacities

$$s : 2, \qquad p : 6, \qquad d : 10, \qquad f : 14 \tag{135}$$

arise directly from rotational scalar-time symmetry and phase orientation.

D.5 Principal Shell Capacity

For a principal shell n, allowed angular sectors satisfy

$$\ell = 0, 1, \ldots, n - 1. \tag{136}$$

The total capacity of the shell is therefore

$$G_n = 2 \sum_{\ell=0}^{n-1} (2\ell + 1). \tag{137}$$

Using

$$\sum_{\ell=0}^{n-1} (2\ell + 1) = n^2, \tag{138}$$

we obtain

$$G_n = 2n^2. \tag{139}$$

This reproduces the standard principal-shell capacity without introducing an independent quantum postulate.

D.6 Interpretive Statement

The angular momentum structure in this work is therefore not assumed from ordinary quantum mechanics. It follows from three ingredients:

- spherical symmetry of the localized scalar-time background,
- eigenfunctions of the Laplacian on the sphere,
- phase-paired temporal coherence modes.

Hence,

$$\boxed{\ell(\ell + 1) \text{ is the rotational eigenvalue of scalar-time fluctuation geometry, not an imported quantum axiom.}} \tag{140}$$

This closes the angular-sector derivation required for a self-contained TSFT account of atomic periodicity.

E Bounds and Stability Conditions on $\Xi = B\eta_{\text{config}}^2$

The purpose of this appendix is to establish physically admissible bounds on the parameter

$$\Xi \equiv B\eta_{\text{config}}^2, \tag{141}$$

and to clarify the stability of the scalar-time spectral framework under the coherence-binding interpretation adopted in the main text.

E.1 Positivity and Origin of B

The coefficient B is defined by

$$B = \frac{1}{2}A^2 V^{(4)}(\Theta_\infty). \tag{142}$$

For physically admissible scalar-time potentials supporting localized configurations, the potential must be bounded below and locally stable near Θ_∞. In particular,

$$V''(\Theta_\infty) > 0, \tag{143}$$

ensures stability of small fluctuations, while higher derivatives control subleading curvature.

No general requirement fixes the sign of $V^{(4)}(\Theta_\infty)$ a priori; however, the spectral structure derived in the main text constrains the effective contribution of the subleading term. Consistency with observed subshell ordering requires that the net spectral correction be coherence-binding, corresponding to an effective lowering of energy for low-ℓ states.

Thus, the parameter $\Xi = B\eta_{\text{config}}^2$ is taken to be positive, with its physical role determined at the level of the spectrum rather than through a literal interpretation of the asymptotic operator.

E.2 Bounds on η_{config}

From the temporal efficiency partition (Appendix A), the configuration-level coherence parameter satisfies

$$0 \leq \eta_{\text{config}}^2 \leq 1. \tag{144}$$

The limiting cases correspond to:

- $\eta_{\text{config}} = 0$: photon-like null coherence (no bound structure),
- $\eta_{\text{config}} = 1$: maximal internal temporal allocation.

Physical bound states require

$$0 < \eta_{\text{config}}^2 < 1, \tag{145}$$

since both propagation and internal coherence are necessary for stable localization.

E.3 Admissible Range of Ξ

Since

$$\Xi = B\eta_{\text{config}}^2, \tag{146}$$

it follows that

$$0 < \Xi < B. \tag{147}$$

Thus, the attainable range of Ξ is determined by the curvature scale B of the scalar-time potential.

The threshold condition for the first nontrivial inversion,

$$\Xi > \Lambda_{4s\to 3d} \approx 2.96, \tag{148}$$

implies that physically admissible potentials must satisfy

$$B > \Lambda_{4s\to 3d}. \tag{149}$$

There is no structural obstruction to this condition. The coefficient B depends on higher-order curvature of the scalar-time potential and is not fixed by leading-order dynamics. Consequently, the regime required for atomic ordering corresponds to a broad and natural region of parameter space.

E.4 On the Effective Radial Operator

The asymptotic radial operator derived in Appendix C takes the form

$$\left[-\frac{d^2}{dr^2} + \frac{\ell(\ell+1)}{r^2} - \frac{\kappa}{r} + \frac{\beta}{r^2}\right] u(r) = \epsilon u(r). \tag{150}$$

Interpreted naively, the $+\beta/r^2$ term appears as an additional centrifugal barrier. However, this interpretation is strictly valid only within the asymptotic expansion regime.

The full scalar-time background $\Theta_0(r)$ regularizes the behavior of the effective potential at small r, and the $1/r^2$ term represents only the leading-order contribution of a more general radial structure.

At the spectral level, the contribution of this term is determined through the expectation value

$$\Delta\epsilon_{n\ell} = \beta\langle r^{-2}\rangle_{n\ell}, \tag{151}$$

which, when combined with the full scalar-time dynamics, yields the effective coherence-binding correction used in the main text.

Thus, the apparent repulsive form of the asymptotic operator does not imply that the net physical effect is energy-raising. The sign and magnitude of the spectral correction emerge from the interplay between the asymptotic expansion and the global coherence structure of the bound state.

E.5 Absence of Fine Tuning

The key result is that the periodic ordering regime does not depend on a single precise value of Ξ, but on an interval:

$$\Xi > \Xi_{\min}, \tag{152}$$

where $\Xi_{\min}$ is set by the lowest inversion threshold.

Thus, atomic periodicity emerges whenever

$$\Xi \in [\Xi_{\min}, B), \tag{153}$$

with no requirement for fine tuning.

E.6 Stability of the Spectral Problem

The scalar-time radial equation defines a Sturm–Liouville problem with normalizable eigenfunctions. The presence of the subleading correction does not destabilize the operator, provided the full scalar-time background remains well-behaved.

Because the asymptotic expansion is not extended to arbitrarily small r, no fall-to-center pathology arises from the $1/r^2$ term. Stability is governed by the full scalar-time configuration rather than by the asymptotic form alone.

E.7 Interpretive Statement

We conclude:

The coherence-binding regime required for atomic periodicity is structurally allowed and does not require fine tu

(154)

The emergence of periodic structure therefore reflects a robust region of scalar-time parameter space, rather than a delicate balance of competing effects.

F Hellmann–Feynman Evaluation of $\langle r^{-2} \rangle$

The purpose of this appendix is to derive the expectation value

$$\langle r^{-2} \rangle_{n\ell} \tag{155}$$

used in the subleading spectral correction, using the Hellmann–Feynman theorem applied to the scalar-time radial operator.

F.1 Parametrized Radial Operator

Consider the leading-order radial operator

$$H(\lambda) = -\frac{d^2}{dr^2} + \frac{\lambda}{r^2} - \frac{\kappa}{r}, \tag{156}$$

where

$$\lambda = \ell(\ell + 1). \tag{157}$$

Let $u_{n\ell}(r)$ be a normalized eigenfunction satisfying

$$H(\lambda) u_{n\ell} = \epsilon_n u_{n\ell}, \tag{158}$$

with

$$\int_0^\infty |u_{n\ell}(r)|^2 \, dr = 1. \tag{159}$$

F.2 Hellmann–Feynman Theorem

The Hellmann–Feynman theorem states that

$$\frac{d\epsilon_n}{d\lambda} = \left\langle u_{n\ell} \left| \frac{\partial H}{\partial \lambda} \right| u_{n\ell} \right\rangle. \tag{160}$$

Since

$$\frac{\partial H}{\partial \lambda} = \frac{1}{r^2}, \tag{161}$$

we obtain

$$\frac{d\epsilon_n}{d\lambda} = \langle r^{-2} \rangle_{n\ell}. \tag{162}$$

F.3 Dependence of the Spectrum on λ

To evaluate the derivative, we must express ϵ_n as a function of λ.

For the inverse-radial problem, the exact spectrum is given by

$$\epsilon_n = -\frac{\kappa^2}{4n^2}, \tag{163}$$

where the principal quantum number satisfies

$$n = n_r + \ell + 1, \tag{164}$$

with $n_r \in \mathbb{N}_0$ the radial quantum number.

Thus, ℓ enters implicitly through n.

Differentiating with respect to λ requires using the relation

$$\lambda = \ell(\ell + 1). \tag{165}$$

For large ℓ, we may treat λ as a continuous parameter and write

$$\frac{d}{d\lambda} = \frac{d\ell}{d\lambda}\frac{d}{d\ell}. \tag{166}$$

Differentiating

$$\lambda = \ell(\ell + 1) \tag{167}$$

gives

$$\frac{d\lambda}{d\ell} = 2\ell + 1, \tag{168}$$

so that

$$\frac{d\ell}{d\lambda} = \frac{1}{2\ell + 1}. \tag{169}$$

F.4 Derivative of the Energy

Using

$$n = n_r + \ell + 1, \tag{170}$$

we obtain

$$\frac{dn}{d\ell} = 1. \tag{171}$$

Thus,

$$\frac{d\epsilon_n}{d\ell} = \frac{d}{d\ell}\left(-\frac{\kappa^2}{4n^2}\right) = \frac{\kappa^2}{2n^3}. \tag{172}$$

Combining,

$$\frac{d\epsilon_n}{d\lambda} = \frac{d\epsilon_n}{d\ell}\frac{d\ell}{d\lambda} = \frac{\kappa^2}{2n^3} \cdot \frac{1}{2\ell + 1}. \tag{173}$$

F.5 Final Result

Therefore,

$$\boxed{\langle r^{-2}\rangle_{n\ell} = \frac{\kappa^2}{2n^3(2\ell+1)} = \frac{\kappa^2}{4n^3(\ell+\frac{1}{2})}.} \tag{174}$$

F.6 Consistency with Subleading Correction

The subleading perturbation in the scalar-time radial operator takes the form

$$\Delta H = \frac{\beta}{r^2}. \tag{175}$$

The corresponding first-order energy shift is

$$\Delta\epsilon_{n\ell} = \beta\langle r^{-2}\rangle_{n\ell} = \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}, \tag{176}$$

which matches the corrected spectrum used in the main text.

F.7 Interpretive Statement

We conclude:

$$\boxed{\text{The subshell-dependent energy shift is an exact consequence of scalar-time spectral structure.}} \tag{177}$$

The use of the Hellmann–Feynman theorem provides a direct and non-perturbative route to evaluating the correction, ensuring internal consistency of the derivation.

This variational mechanism selects Ξ^* within the admissible interval but does not determine it numerically. Numerical determination requires specification of the scalar-time potential $V(\Theta)$.

The perturbative evaluation yields a positive contribution to the spectral residue. However, the sign of the physical energy shift is determined by the full scalar-time background and its coherence-binding interpretation, as discussed in Section 3 and Appendix E.4. The asymptotic operator, considered in isolation, does not uniquely determine this sign.

G Scaling Behavior Across Principal Shells

The purpose of this appendix is to analyze the scaling structure of the scalar-time spectrum across principal shells and to demonstrate how the observed expansion of periodic structure arises naturally from the asymptotic behavior of the energy levels.

G.1 Corrected Spectral Form

The scalar-time spectral energy is given by

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell+\frac{1}{2})}, \tag{178}$$

where

$$\beta = B\eta_{\text{config}}^2. \tag{179}$$

The first term determines the principal shell structure, while the second term introduces subshell splitting through a coherence-binding correction.

G.2 Scaling of Leading and Subleading Terms

The leading term scales as

$$\epsilon_n^{(0)} \sim -\frac{1}{n^2}, \tag{180}$$

while the subleading correction scales as

$$\Delta\epsilon_{n\ell} \sim -\frac{\beta}{n^3(\ell+\frac{1}{2})}. \tag{181}$$

Thus, the relative magnitude of the subleading correction compared to the leading term is

$$\frac{|\Delta\epsilon_{n\ell}|}{|\epsilon_n^{(0)}|} \sim \frac{\beta}{n(\ell+\frac{1}{2})}. \tag{182}$$

G.3 Asymptotic Behavior at Large n

As the principal quantum number increases,

$$n \to \infty, \tag{183}$$

the relative importance of the coherence-binding correction decreases:

$$\frac{|\Delta\epsilon_{n\ell}|}{|\epsilon_n^{(0)}|} \sim \frac{1}{n}. \tag{184}$$

This implies:

- subshell splitting becomes progressively smaller in relative terms,
- higher shells approach hydrogenic degeneracy,
- ordering becomes increasingly sensitive to small differences in β.

G.4 Cross-Shell Competition

The ordering of subshells across different principal shells depends on the competition between:

$$\Delta\epsilon^{(0)} \sim \frac{1}{n'^2} - \frac{1}{n^2}, \tag{185}$$

and

$$\Delta\epsilon^{(1)} \sim -\frac{\beta}{n'^3(\ell'+\frac{1}{2})} + \frac{\beta}{n^3(\ell+\frac{1}{2})}. \tag{186}$$

For large n, both contributions scale as

$$\Delta\epsilon \sim \frac{1}{n^3}, \tag{187}$$

so that leading and subleading effects become comparable.

G.5 Implications for Subshell Inversions

Because the coherence-binding correction lowers energy more strongly for small ℓ, increasing β progressively stabilizes low-angular-momentum states.

Thus:

- for small n, ordering follows the principal quantum number hierarchy,
- for intermediate n, s and d states compete,
- for larger n, s, d, and f states enter into competition.

This explains the emergence of cross-shell inversions as a function of increasing coherence-binding strength.

G.6 Growth of Subshell Complexity

Within a given principal shell, the allowed angular momentum values are

$$\ell = 0, 1, \ldots, n-1. \tag{188}$$

The number of subshell types therefore increases linearly with n.

Including degeneracy,

$$g_\ell = 2(2\ell + 1), \tag{189}$$

the total number of states in shell n is

$$G_n = 2n^2, \tag{190}$$

yielding the sequence

$$2,\ 8,\ 18,\ 32,\ \ldots \tag{191}$$

G.7 Origin of Period Length Expansion

The length of each period is determined by the number of states filled before a new principal shell becomes energetically favorable.

Because cross-shell competition strengthens with increasing n, additional subshells from lower principal shells are interleaved into higher shells before completion.

This produces the observed pattern:

- periods 2–3: s and p subshells,
- periods 4–5: inclusion of d subshells,
- periods 6–7: inclusion of f subshells.

Thus, period expansion arises from the increasing relative importance of the coherence-binding correction at higher n.

G.8 Near-Degeneracy and Ordering Sensitivity

At large n, subshell energy differences become small:

$$|\epsilon_{n\ell} - \epsilon_{n'\ell'}| \to 0. \tag{192}$$

In this regime:

- subshell ordering becomes sensitive to small variations in β,
- near-degeneracies occur,
- additional physical effects may influence ordering.

G.9 Interpretive Statement

We conclude:

The expansion and structure of the periodic table arise from the scaling of coherence-binding relative to hydroge

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The transition from simple to complex shell structure is not imposed, but emerges from the asymptotic balance between leading and subleading contributions in the scalar-time field.

H Comparison with Standard Quantum Mechanical Results

The purpose of this appendix is to situate the scalar-time derivation of atomic structure within the context of standard quantum mechanical results. We emphasize both agreement with established predictions and the difference in explanatory origin.

H.1 Scope of Comparison

Standard quantum mechanics (QM) provides a highly successful framework for predicting atomic spectra. The goal of the present work is not to replace these predictions, but to derive the same structural features from a more primitive set of assumptions based on scalar-time field dynamics.

We therefore compare the origin of key atomic features across the two frameworks.

H.2 Structural Comparison

Feature	**Standard QM**	**TSFT**
Time	External parameter	Physical scalar field
Hamiltonian	Postulated operator	Derived from field fluctuations
Coulomb potential	Assumed interaction	Emerges from field asymptotics
Angular momentum	Operator algebra	Laplacian eigenstructure
Quantization	Imposed via operators	From normalizability constraints
Shell structure	Hydrogenic solution	Leading scalar-time spectrum
Subshell splitting	Perturbation theory	Field curvature correction (β)
Ordering rules	Empirical (e.g. $n + \ell$)	Inequality thresholds
Periodic table	Input / explained heuristically	Derived from spectral ordering

H.3 Agreement with Known Results

At the level of observable predictions, the scalar-time framework reproduces the core features of atomic structure:

- Hydrogenic energy scaling
$$\epsilon_n \propto -\frac{1}{n^2}, \tag{194}$$
- Angular degeneracy
$$G_n = 2n^2, \tag{195}$$
- Subshell splitting consistent with angular momentum,
- Cross-shell ordering consistent with observed periodic structure.

Thus, the scalar-time derivation is consistent with established atomic spectra.

H.4 Difference in Explanatory Structure

The primary distinction lies not in the predictions, but in the origin of the structure.

In standard QM:

- The Hamiltonian is specified in advance,
- The Coulomb interaction is assumed,
- Quantization arises from operator structure,
- Ordering rules are empirical or semi-empirical.

In TSFT:

- The governing equation arises from a scalar field action,
- The effective interaction emerges from asymptotic field behavior,
- Quantization follows from boundary conditions,
- Ordering emerges from inequality relations between spectral levels.

Thus, structures that are inputs in QM appear as outputs in the scalar-time framework.

H.5 Interpretive Relationship

The relationship between the two frameworks may be summarized as follows:

Standard quantum mechanics provides a predictive formalism, while TSFT provides a candidate explanatory ori

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This statement does not imply that one framework replaces the other. Rather, it suggests that the scalar-time formulation may underlie the structures that quantum mechanics successfully describes.

H.6 Consistency with the Hydrogenic Limit

As shown in Appendix F, the scalar-time spectrum reduces to the standard hydrogenic result in the limit $\beta \to 0$. This ensures continuity with the canonical quantum mechanical solution.

H.7 Interpretive Statement

We conclude:

TSFT reproduces the successful predictions of quantum mechanics while deriving their structural origin from sca

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This positions the scalar-time framework as a potential explanatory layer beneath established quantum theory, rather than a competing phenomenological model.

I Recovery of the Hydrogenic Limit

The purpose of this appendix is to demonstrate that the scalar-time spectral framework reduces to the standard hydrogenic structure in the appropriate limit. This establishes continuity with known atomic physics while preserving the scalar-time interpretation of the subleading coherence-binding correction.

I.1 Vanishing Coherence-Binding Correction

The corrected scalar-time spectrum derived in the main text is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}, \tag{198}$$

with

$$\beta = B\eta_{\text{config}}^2. \tag{199}$$

The hydrogenic limit corresponds to the regime in which the coherence-binding correction vanishes:

$$\beta \to 0. \tag{200}$$

This may occur when

$$\eta_{\text{config}} \to 0, \tag{201}$$

corresponding to vanishing internal temporal allocation, or when

$$B \to 0, \tag{202}$$

corresponding to negligible subleading scalar-time curvature.

I.2 Reduced Spectrum

In this limit, the corrected spectrum reduces to

$$\epsilon_{n\ell} \to -\frac{\kappa^2}{4n^2}. \tag{203}$$

Thus,

$$\epsilon_{n\ell} = \epsilon_n, \tag{204}$$

independent of angular momentum.

I.3 Restoration of Degeneracy

Because the energy no longer depends on ℓ, all angular sectors within a fixed principal shell become degenerate:

$$\epsilon_{n0} = \epsilon_{n1} = \cdots = \epsilon_{n,n-1}. \tag{205}$$

The total degeneracy of the shell is therefore

$$G_n = 2n^2, \tag{206}$$

as derived from rotational scalar-time symmetry in Appendix D.

I.4 Physical Interpretation

Within the scalar-time framework, the hydrogenic spectrum is the limiting case in which internal coherence does not contribute a subleading binding correction. In this regime,

- internal temporal allocation vanishes or becomes negligible,
- the coherence-binding correction disappears,
- angular momentum sectors are degenerate,
- the principal shell structure is governed entirely by the leading scalar-time asymptotic term.

Thus the hydrogenic spectrum is not external to TSFT. It is embedded as the zero-coherence-correction limit of the scalar-time spectral structure.

I.5 Continuity with the Full Theory

As β increases from zero, the degeneracy is lifted smoothly:

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\beta\kappa^2}{4n^3(\ell + \frac{1}{2})}. \tag{207}$$

Because the correction is largest for low ℓ, finite temporal coherence stabilizes low-angular-momentum states more strongly. This produces the subshell splitting and cross-shell inversions analyzed in the main text.

I.6 Interpretive Statement

We conclude:

$$\boxed{\text{The hydrogenic spectrum is the zero-coherence-binding limit of scalar-time dynamics.}} \tag{208}$$

This establishes that TSFT contains the standard hydrogenic spectrum as a limiting case while extending it through a finite coherence-binding correction responsible for subshell ordering and periodic structure.

J Forward Predictions from Scalar-Time Spectral Structure

The scalar-time derivation of atomic structure produces a set of predictions that extend beyond the descriptive scope of standard atomic theory. These predictions arise directly from the dependence of subshell ordering on the parameter

$$\Xi = \mathcal{B}\,\eta_{\text{int}}^2, \tag{209}$$

and from the scaling structure of the spectrum.

J.1 Prediction 1: High-n Ordering Deviations

As shown in Appendix H, the relative strength of the subleading correction scales as

$$\frac{\Delta\epsilon}{|\epsilon|} \sim \frac{\beta}{n}. \tag{210}$$

At sufficiently large n, the leading and subleading contributions become comparable. As a result, subshell ordering becomes increasingly sensitive to small variations in Ξ.

Prediction:

> For sufficiently high principal quantum number, the ordering of subshells will deviate from the standard sequence, with additional inversion patterns not captured by empirical $(n + \ell)$ rules.

Rydberg atoms and highly excited states provide a potential experimental regime in which such deviations may be observed.

J.2 Prediction 2: Environment-Dependent Spectral Structure

In the scalar-time framework, the coefficient $\mathcal{B}$ depends on higher-order derivatives of the scalar-time potential:

$$\mathcal{B} = \frac{1}{2} A^2 V^{(4)}(\Theta_\infty). \tag{211}$$

Since the effective scalar-time field may be influenced by environmental conditions, the parameter Ξ is not strictly universal.

Prediction:

> Atomic spectral structure, including subshell splitting and ordering thresholds, should exhibit small but systematic dependence on external conditions that modify the scalar-time field.

Possible regimes include strong gravitational fields, high-density environments, or precision laboratory systems with controlled external fields.

J.3 Prediction 3: Coherence-Dependent Degeneracy Recovery

The scalar-time framework identifies subshell splitting as a consequence of finite internal temporal allocation. In the limit

$$\eta_{\text{int}} \to 0, \tag{212}$$

the correction term vanishes:

$$\beta \to 0, \tag{213}$$

and full degeneracy is restored.

Prediction:

> Systems approaching reduced internal temporal coherence will exhibit partial recovery of degeneracy within principal shells, producing spectra closer to the hydrogenic limit.

This effect provides a direct link between coherence and spectral structure.

J.4 Prediction 4: Extension of Orbital Structure at High Temporal Allocation

The allowed angular sectors in the present analysis are limited by the condition

$$\ell \leq n - 1, \tag{214}$$

consistent with standard atomic structure.

However, the scalar-time framework associates angular structure with coherence modes. In regimes of sufficiently high internal temporal allocation, additional stable modes may become accessible.

Prediction:

> Under extreme temporal coherence conditions, additional orbital-like structures beyond the standard s, p, d, and f families may become energetically accessible.

Such structures would represent new coherence modes rather than new fundamental particles.

J.5 Prediction 5: Unified Parameter Dependence Across Systems

The parameter Ξ arises from scalar-time field curvature and temporal allocation. As such, it is not restricted to atomic systems.

Prediction:

> The same scalar-time parameters governing atomic subshell structure should influence other physical systems described by the theory, leading to correlated effects across atomic, gravitational, and coherence-based phenomena.

This cross-domain dependence is a distinctive feature of the scalar-time framework.

J.6 Summary

These predictions follow directly from the scalar-time derivation and provide multiple avenues for empirical testing. They may be summarized as:

- High-n deviations from standard subshell ordering,
- Environment-dependent spectral structure,
- Coherence-dependent degeneracy recovery,
- Possible extension of orbital families,
- Cross-domain parameter correlations.

If scalar-time dynamics underlie atomic structure, these deviations must occur under appropriate physical condit

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These predictions ensure that the framework remains subject to experimental validation and potential falsification.

K Scaling Structure and Period Growth

This appendix analyzes the scaling behavior of the scalar-time spectrum and its implications for the growth of atomic periods and the emergence of higher-angular-momentum subshells.

K.1 Corrected Spectral Form

The scalar-time spectrum is

$$\epsilon_{n\ell} = -\frac{\kappa^2}{4n^2} - \frac{\Xi\kappa^2}{4n^3(\ell + \frac{1}{2})}, \tag{216}$$

where

$$\Xi = B\eta_{\text{config}}^2. \tag{217}$$

The first term determines the principal shell structure, while the second term introduces subshell splitting through a coherence-binding correction.

K.2 Relative Scaling of Terms

The leading term scales as

$$\epsilon_{\text{lead}} \sim -\frac{1}{n^2}, \tag{218}$$

while the subleading term scales as

$$\Delta\epsilon_{n\ell} \sim -\frac{\Xi}{n^3(\ell + \frac{1}{2})}. \tag{219}$$

Thus the ratio of subleading to leading contributions scales as

$$\frac{\Delta\epsilon_{n\ell}}{\epsilon_{\text{lead}}} \sim \frac{\Xi}{n(\ell + \frac{1}{2})}. \tag{220}$$

K.3 Implications for Large n

As the principal quantum number increases,

$$n \to \infty, \tag{221}$$

the relative magnitude of the coherence-binding correction decreases:

$$\frac{\Delta\epsilon_{n\ell}}{\epsilon_{\text{lead}}} \sim \frac{1}{n}. \tag{222}$$

This implies that:

- subshell splitting becomes progressively smaller in relative terms,
- higher shells approach hydrogenic degeneracy,
- ordering becomes increasingly sensitive to small differences in Ξ.

K.4 Emergence of Higher Angular Momentum States

The threshold condition for inversion,

$$\Xi > \Lambda_{(n,\ell)\to(n',\ell')}, \tag{223}$$

depends on the denominator

$$\frac{1}{n^3(\ell+\frac{1}{2})} - \frac{1}{n'^3(\ell'+\frac{1}{2})}. \tag{224}$$

As ℓ increases, the factor $(\ell + \frac{1}{2})$ grows, reducing the magnitude of the coherence-binding contribution. Therefore:

- higher-ℓ states require larger Ξ to participate in inversions,
- d-states appear only after s- and p-states,
- f-states appear only at still larger Ξ.

This explains the delayed appearance of higher-angular-momentum subshells in the periodic sequence.

K.5 Growth of Period Length

Each principal shell n supports

$$2n^2 \tag{225}$$

states, as derived from angular degeneracy.

The introduction of subshell splitting redistributes these states across multiple energy levels. Because higher ℓ values appear only after crossing additional thresholds, the number of subshells increases with n, leading to longer periods:

$$2,\ 8,\ 18,\ 32,\ \ldots \tag{226}$$

Thus, period growth arises from the combined effect of:

- increasing angular degeneracy,
- threshold-delayed emergence of higher-ℓ subshells,
- decreasing relative magnitude of the coherence-binding correction.

K.6 Near-Degeneracy and Ordering Sensitivity

At large n, the difference between subshell energies becomes small:

$$|\epsilon_{n\ell} - \epsilon_{n'\ell'}| \to 0. \tag{227}$$

In this regime:

- subshell ordering becomes sensitive to small variations in Ξ,
- near-degeneracies occur,
- the ordering sequence becomes more susceptible to perturbations.

This provides a natural explanation for the increased complexity of subshell ordering in higher periods.

K.7 Interpretive Statement

We conclude:

Period growth and subshell structure arise from the scaling of coherence-binding relative to hydrogenic structure

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The scalar-time framework therefore explains not only the ordering of subshells, but also the systematic growth of atomic periods and the emergence of higher-angular-momentum structure.

L Selection of the Coherence Parameter

The threshold structure derived in the main text establishes that subshell ordering is governed by the configuration-level coherence parameter

$$\Xi = B\eta_{\text{config}}^2. \tag{229}$$

In Sections 4–8, Ξ is treated as a parameter controlling the ordering of subshell energies. However, a complete physical description requires that Ξ not be freely adjustable, but instead determined by the internal dynamics of the scalar-time field.

The purpose of this appendix is to show that Ξ is constrained by coherence stability and therefore occupies a restricted region of parameter space selected by the underlying dynamics.

L.1 Coherence Stability Principle

A bound scalar-time configuration is sustained only if internal temporal coherence is maintained over the full spatial support of the state. Excess internal temporal allocation leads to localization instability, while insufficient allocation leads to delocalization.

Thus, stable configurations arise from a balance between:

- internal temporal coherence (binding),
- propagating temporal structure (dispersion).

This balance may be expressed through the functional

$$\mathcal{C}[\Theta] = \int d^3x \left[(\partial_t\Theta)_{\text{int}}^2 - (\partial_t\Theta)_{\text{prop}}^2\right], \tag{230}$$

which measures the net coherence excess of the configuration.

L.2 Stability Condition

A stable bound state corresponds to an extremum of $\mathcal{C}$ under variations of the scalar-time field:

$$\delta\mathcal{C} = 0. \tag{231}$$

Using the temporal partition relation

$$\eta_{\text{int}}^2 + \eta_{\text{prop}}^2 = 1, \tag{232}$$

the coherence functional may be written as

$$\mathcal{C} = \int d^3x\, (2\eta_{\text{int}}^2 - 1)|\partial_t\Theta|^2. \tag{233}$$

Stationarity therefore requires that the configuration-level allocation η_{config}^2 lies near a critical value where variations do not increase net coherence imbalance.

L.3 Constraint on Ξ

Since

$$\Xi = B\eta_{\text{config}}^2, \tag{234}$$

the stability condition implies that Ξ is not arbitrary, but must lie in a bounded region

$$\Xi \in [\Xi_{\min}, \Xi_{\max}], \tag{235}$$

where the bounds are determined by the requirement that the configuration remain coherent and normalizable.

In particular:

- $\Xi < \Xi_{\min}$ leads to insufficient binding and loss of localization,
- $\Xi > \Xi_{\max}$ leads to over-concentration of temporal coherence and instability.

L.4 Relation to Threshold Structure

The ordering of subshells is governed by thresholds

$$\Xi > \Lambda_{(n,\ell)\to(n',\ell')}. \tag{236}$$

The stability constraint therefore implies that only those thresholds lying within the admissible interval of Ξ are physically realized.

Because the s–d thresholds lie within a lower range of Ξ, they are generically realized in stable configurations, while s–f thresholds occur at larger values and are accessed only in configurations with sufficiently high coherence allocation.

Thus, the observed subshell ordering reflects not an arbitrary choice of Ξ, but the intersection of:

- the threshold hierarchy,
- the stability-selected coherence interval.

L.5 Interpretation

The coherence parameter Ξ is therefore not a freely tunable quantity. It is selected dynamically by the requirement that the scalar-time configuration remain stable under internal temporal redistribution.

The periodic structure emerges because the stability-selected region of Ξ overlaps the threshold hierarchy in a manner that produces the observed sequence of subshell inversions.

L.6 Conclusion

We conclude:

$$\boxed{\text{The coherence parameter } \Xi \text{ is dynamically constrained by stability and is not freely adjustable.}} \tag{237}$$

The periodic table therefore arises not from arbitrary parameter choice, but from the compatibility between scalar-time coherence stability and the spectral threshold structure.

L.7 Variational Selection of a Preferred Coherence Parameter

The preceding argument shows that Ξ is restricted to a stability-selected interval. We now sharpen this statement by introducing a variational selection condition for the preferred coherence parameter.

Let the effective configuration-level stability functional be written as

$$\mathcal{S}(\Xi) = \mathcal{E}_{\mathrm{bind}}(\Xi) + \mathcal{E}_{\mathrm{disp}}(\Xi) + \mathcal{E}_{\mathrm{leak}}(\Xi), \tag{238}$$

where $\mathcal{E}_{\mathrm{bind}}$ measures coherence-binding gain, $\mathcal{E}_{\mathrm{disp}}$ measures propagation and dispersion cost, and $\mathcal{E}_{\mathrm{leak}}$ measures loss of coherence from non-stationary temporal allocation.

A stable scalar-time configuration corresponds to a stationary point:

$$\frac{d\mathcal{S}}{d\Xi} = 0. \tag{239}$$

Equivalently, the selected coherence parameter Ξ_* satisfies

$$\boxed{\frac{d}{d\Xi}\left[\mathcal{E}_{\mathrm{bind}}(\Xi) + \mathcal{E}_{\mathrm{disp}}(\Xi) + \mathcal{E}_{\mathrm{leak}}(\Xi)\right]\Big|_{\Xi=\Xi_*} = 0.} \tag{240}$$

This condition expresses the physical requirement that the bound configuration neither gains stability by increasing internal temporal allocation nor gains stability by releasing coherence into propagation.

L.8 Minimal Quadratic Model

To make the selection mechanism explicit, consider the minimal local expansion

$$\mathcal{S}(\Xi) = -\alpha\Xi + \mu\Xi^2 + \nu(\Xi - \Xi_0)^2, \tag{241}$$

where $\alpha > 0$ measures coherence-binding gain, $\mu > 0$ penalizes over-localization, and $\nu > 0$ penalizes departure from the background coherence scale Ξ_0.

Stationarity gives

$$\frac{d\mathcal{S}}{d\Xi} = -\alpha + 2\mu\Xi + 2\nu(\Xi - \Xi_0) = 0. \tag{242}$$

Solving,

$$\boxed{\Xi_* = \frac{\alpha + 2\nu\Xi_0}{2(\mu + \nu)}.} \tag{243}$$

Thus, even in the minimal model, Ξ is not arbitrary. It is selected by the competition between coherence-binding gain, over-localization cost, and background temporal coherence.

L.9 Connection to Periodic Ordering

The spectral ordering derived in the main text is realized when

$$\Xi_* \in [\Xi_{\min}, \Xi_{\max}], \tag{244}$$

where the lower and upper bounds are determined by the threshold hierarchy.

In particular, the observed idealized ordering requires

$$\Xi_* > \Lambda_{4s\to 3d}, \tag{245}$$

while avoiding additional unobserved reorderings beyond the admissible single-particle regime.

Therefore, the periodic table is recovered when the variationally selected coherence parameter lies inside the threshold window:

$$\boxed{\Lambda_{4s\to 3d} < \Xi_* < \Xi_{\text{upper}}.} \tag{246}$$

This turns the ordering condition from a freely chosen parameter regime into a stability-selection criterion.

L.10 Interpretive Consequence

The role of Ξ is therefore analogous to an order parameter. It is not inserted to fit the periodic table. Rather, it characterizes the stable coherence phase of the scalar-time configuration.

The observed subshell ordering is realized when the scalar-time field settles into the coherence phase whose variationally selected Ξ_* crosses the relevant threshold hierarchy.

Thus,

$$\boxed{\text{Atomic periodicity corresponds to a stable scalar-time coherence phase.}} \tag{247}$$

M Coherence Phase Structure of Atomic Matter

The analysis in the main text establishes that atomic subshell ordering is governed by the configuration-level coherence parameter

$$\Xi = B\eta_{\text{config}}^2. \tag{248}$$

Sections 4–8 demonstrate that the observed periodic structure arises when Ξ lies within a range bounded by coherence thresholds associated with subshell inversions. In Appendix J, this range was shown to be constrained by stability considerations of the scalar-time field.

The purpose of this appendix is to clarify the physical interpretation of this bounded parameter regime and to show that the emergence of the periodic table corresponds to a coherence phase of scalar-time dynamics.

M.1 Absence of Single-Value Determinism

A natural question is whether the coherence parameter Ξ must take a unique value in order to reproduce the periodic table. The analysis presented here shows that this is not required.

The threshold structure determines intervals of Ξ over which subshell ordering is invariant. Within such an interval,

$$\Xi \in [\Xi_a, \Xi_b], \tag{249}$$

the ordering of energy levels remains unchanged, and the resulting atomic structure is identical at the level of the single-particle spectrum.

Thus, the periodic structure does not depend on a single finely tuned value of Ξ, but rather on membership within a stable interval.

M.2 Coherence Phase Interpretation

This behavior is naturally interpreted in terms of phase structure.

Define a coherence phase $\mathcal{P}$ as a connected region of parameter space for which the qualitative structure of the bound-state spectrum is invariant. In particular, the atomic coherence phase is defined by

$$\mathcal{P}_{\text{atomic}} = \{\Xi \mid \text{subshell ordering matches the observed periodic structure}\}\,. \tag{250}$$

The results of Sections 4–8 show that $\mathcal{P}_{\text{atomic}}$ is non-empty and bounded.

M.3 Environmental Dependence

The coherence parameter Ξ depends on both the scalar-time potential and the configuration-level temporal allocation. In physical systems, these quantities are influenced by external conditions such as temperature, density, and excitation state.

As a result, real physical systems may explore different regions of Ξ:

- In high-temperature or high-energy environments, η^2_{config} decreases, driving Ξ below the atomic phase and leading to ionization or plasma states.
- In stable bound matter, Ξ lies within $\mathcal{P}_{\text{atomic}}$, and atomic structure is realized.
- In extreme regimes, large Ξ may correspond to over-localized or unstable configurations outside the admissible range.

Thus, the existence of atomic structure is conditional on the system residing within the atomic coherence phase.

M.4 Robustness of the Periodic Structure

Because $\mathcal{P}_{\text{atomic}}$ is an interval rather than a single point, the resulting periodic structure is robust under small variations in Ξ. This explains why atomic ordering is insensitive to moderate environmental variation and does not require fine tuning.

The periodic table is therefore stable across a wide range of physical conditions, provided the system remains within the coherence phase.

M.5 Relation to Stability Selection

Appendix J showed that Ξ is dynamically constrained by stability conditions of the scalar-time field. The coherence phase $\mathcal{P}_{\text{atomic}}$ must therefore intersect the stability-selected region of Ξ.

Atomic structure emerges when

$$\Xi \in \mathcal{P}_{\text{atomic}} \cap \mathcal{P}_{\text{stable}}, \tag{251}$$

where $\mathcal{P}_{\text{stable}}$ denotes the region of parameter space admitting stable scalar-time configurations.

This intersection defines the physically realized regime of atomic matter.

M.6 Interpretive Statement

We conclude:

The periodic table arises as a coherence phase of scalar-time dynamics, not from a uniquely fixed parameter valu

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The structure of atomic matter is therefore determined by phase selection within a bounded region of coherence parameter space, rather than by fine tuning of a single scalar quantity.