Particle Mass Emergence from Scalar-Time Coherence in Time-Scalar Field Theory

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Abstract

Time-Scalar Field Theory (TSFT) promotes time to a fundamental scalar field $\Theta(x, t)$ governing information flow and coherence structure. Previous work established the existence of stable scalar-time excitations, rivet structures, and discrete spectral families arising from viability-constrained scalar-time dynamics. However, particle mass prediction remained incomplete due to the absence of a first-principles derivation linking scalar-time coherence spectra to relativistic mass.

In this work, we derive particle masses directly from scalar-time coherence dynamics. Starting from the scalar-time field $\Theta(x, t)$, we construct a viability-constrained coherence functional whose stationary points define a self-adjoint scalar-time operator. The resulting eigenvalue spectrum generates a discrete scalar-time frequency ladder. We then derive the relativistic propagation equation for scalar-time excitations and demonstrate that the coherence eigenvalues appear as Klein–Gordon mass-shell terms. This yields the fundamental relation

$$m_n^2 = \lambda_n$$

where λ_n are scalar-time coherence eigenvalues. Particle families, spins, and charges emerge from previously derived rivet closure and holonomy structures, while masses arise directly from the spectral ladder.

Importantly, particle identities are not assumed. Instead, the theory produces an unlabeled catalog of admissible scalar-time coherence states. Known particles are identified only through post hoc comparison, while unmatched states constitute predictions. This establishes a non-circular, first-principles pathway from scalar-time dynamics to particle spectrum emergence.

1 Introduction

The Standard Model of particle physics successfully describes known elementary particles and their interactions, yet it does not derive particle masses from first principles. Instead, particle masses arise through phenomenological parameters introduced via symmetry breaking mechanisms. This leaves open the fundamental question of why the particle spectrum takes its observed form. Observed particle properties and current search limits are taken relative to the standard summaries of the Particle Data Group [16]. In particular, any extension of the scalar-time spectrum beyond the observed fermion sector must ultimately be compared against existing collider bounds on heavy charged and neutral leptons [18, 19]. We also note that the scalar-time spectral ladder derived here is conceptually distinct from conventional Kaluza–Klein towers arising from compact extra spatial dimensions [17].

Time-Scalar Field Theory (TSFT) provides a framework in which time is promoted from a parameter to a fundamental scalar field,

$$\Theta = \Theta(x,t),$$

governing information propagation, coherence, and stability. In this framework, physical structures emerge as stable coherence configurations within the scalar-time field. Previous work established that localized scalar-time excitations form stable rivet structures, and that viability-constrained dynamics produce discrete spectral families.

These results suggest that particle structure arises from scalar-time coherence rather than being fundamental. However, a first-principles derivation of particle masses has remained incomplete. In particular, while earlier work identified spectral ladders and admissible families, the connection between scalar-time eigenvalues and relativistic mass had not yet been derived.

In this work, we complete this connection. Starting from scalar-time coherence dynamics, we construct a viability-constrained coherence functional whose stationary points define a self-adjoint operator governing scalar-time excitations. The resulting eigenvalue spectrum produces a discrete scalar-time frequency ladder. We then demonstrate that these eigenvalues appear directly as relativistic mass-shell terms in the propagation equation for scalar-time excitations.

The central result of this work is the relation

$$m_n^2 = \lambda_n,$$

where λ_n are eigenvalues of the scalar-time coherence operator. This establishes particle mass as an emergent property of scalar-time stability.

Importantly, particle identities are not assumed in advance. Instead, the theory generates an unlabeled catalog of admissible scalar-time states characterized by spectral index, holonomy structure, and stability classification. Known particles are identified only through post hoc comparison, while unmatched states constitute predictions.

This approach avoids circular reasoning and provides a non-phenomenological pathway from scalar-time dynamics to particle spectrum emergence. The result is a first-principles derivation of particle masses and families within the TSFT framework.

2 Time-Scalar Field Foundations

Time-Scalar Field Theory (TSFT) promotes time from a background parameter to a physical scalar field,

$$\Theta = \Theta(x,t),$$

which governs information propagation and coherence structure. In this framework, spatial relationships emerge from information delays, and physical structures arise from stable scalar-time configurations.

The fundamental dynamical quantity in TSFT is the scalar-time gradient,

$$\partial_\mu \Theta,$$

which determines coherence flow and stability properties. Regions of stable scalar-time coherence form localized structures that persist under perturbations. These structures correspond to physical entities within the TSFT framework.

We consider perturbations around a background scalar-time field,

$$\Theta(x,t) = \Theta_0(x,t) + \psi(x,t),$$

where Θ_0 represents the background scalar-time configuration and $\psi(x,t)$ denotes localized scalar-time excitations. Stable excitations correspond to persistent structures satisfying boundedness and stability requirements.

Persistence of scalar-time excitations requires that the total coherence remain bounded over time. This condition may be expressed as

$$\frac{d}{dt}\int |\psi(x,t)|^2 \, d^3x = 0,$$

which implies stationary coherence configurations. These stationary solutions define candidate particle-like structures within TSFT.

Previous work established that stable scalar-time excitations form discrete rivet structures. These rivet structures arise from the balance between dispersion and collapse within scalar-time dynamics. Dispersion increases coherence leakage, while collapse increases gradient cost. Stable configurations therefore occur at stationary points of scalar-time coherence dynamics.

These stability conditions naturally lead to spectral structure. Specifically, stable scalar-time excitations satisfy eigenvalue equations of the form

$$C_\Theta \psi = \lambda \psi,$$

where C_Θ is the scalar-time coherence operator and λ represents coherence eigenvalues. These eigenvalues define discrete scalar-time frequencies associated with persistent structures.

The derivation of the coherence operator and resulting spectral ladder forms the basis for particle mass emergence within TSFT.

3 Viability-Constrained Coherence Functional

Stable scalar-time excitations arise from the requirement that coherence remains localized and persistent. This persistence requirement leads naturally to a coherence functional whose stationary points correspond to stable structures.

We define the scalar-time coherence functional

$$\mathcal{C}[\psi] = \int_\Omega \left[\zeta \frac{|\partial_\Theta \psi|^2}{\rho_\Theta(\Theta)} + \frac{\eta |\psi|^2}{\rho_\Theta(\Theta) - \epsilon} + \frac{|\psi|^2}{\sqrt{\rho_\Theta(\Theta)}} \right] d\Theta,$$

where $\rho_\Theta(\Theta)$ represents scalar-time density, ϵ is the viability threshold, and ζ and η are structural coefficients determined by scalar-time dynamics.

The first term penalizes rapid oscillations and represents coherence gradient cost. The second term penalizes configurations approaching the viability threshold, enforcing stability constraints. The third term penalizes low-density persistence, ensuring localization of scalar-time excitations.

Stable scalar-time structures correspond to stationary points of the coherence functional subject to normalization:

$$\int_{\Omega} |\psi(\Theta)|^2 d\Theta = 1.$$

We therefore define the constrained functional

$$\mathcal{J}[\psi] = \mathcal{C}[\psi] - \lambda \left(\int_{\Omega} |\psi|^2 d\Theta - 1 \right),$$

where λ is a Lagrange multiplier enforcing normalization.
Stationarity requires

$$\delta \mathcal{J} = 0.$$

Variation with respect to ψ^* yields the Euler–Lagrange equation

$$-\zeta \partial_\Theta \left(\frac{1}{\rho_\Theta(\Theta)} \partial_\Theta \psi \right) + \left[\frac{\eta}{\rho_\Theta(\Theta) - \epsilon} + \frac{1}{\sqrt{\rho_\Theta(\Theta)}} \right] \psi = \lambda \psi.$$

This defines the scalar-time coherence operator

$$C_\Theta = -\zeta \partial_\Theta \left(\frac{1}{\rho_\Theta(\Theta)} \partial_\Theta \right) + \frac{\eta}{\rho_\Theta(\Theta) - \epsilon} + \frac{1}{\sqrt{\rho_\Theta(\Theta)}}.$$

Stable scalar-time excitations therefore satisfy the eigenvalue equation

$$C_\Theta \psi_n = \lambda_n \psi_n.$$

The eigenvalues λ_n define discrete scalar-time coherence frequencies. These frequencies form the basis for particle mass emergence in the TSFT framework.

4 Compact Viability Domain

The scalar-time coherence operator derived in the previous section admits a discrete spectrum only if the admissible scalar-time domain is compact. This compactness arises naturally from the viability constraint imposed by scalar-time persistence.

We define the viability condition

$$\rho_\Theta(\Theta) \geq \epsilon,$$

where ϵ represents the minimum scalar-time density required for persistent coherence. Regions where $\rho_\Theta(\Theta) < \epsilon$ do not support stable scalar-time excitations and are therefore excluded from the admissible domain.

In addition, we assume that scalar-time density decays asymptotically,

$$\rho_\Theta(\Theta) \to 0 \quad \text{as} \quad |\Theta| \to \infty.$$

This behavior is physically natural, as scalar-time coherence weakens far from stable configurations. Together with the viability constraint, this condition ensures that the admissible domain

$$\Omega = \{\Theta \,|\, \rho_\Theta(\Theta) \geq \epsilon\}$$

is bounded.

The boundedness of Ω implies compactness of the scalar-time domain. Under these conditions, the scalar-time coherence operator C_Θ is self-adjoint on a compact domain, and therefore admits a discrete spectrum.

By the spectral theorem, the eigenvalue problem

$$C_\Theta \psi_n = \lambda_n \psi_n$$

produces a countable set of discrete eigenvalues,

$$\lambda_1 \leq \lambda_2 \leq \lambda_3 \leq \cdots .$$

These eigenvalues define the scalar-time coherence ladder. Each eigenvalue corresponds to a stable scalar-time excitation and therefore to a candidate particle state.

The compact viability domain thus guarantees the existence of a discrete scalar-time spectrum, forming the foundation for particle mass emergence in TSFT.

5 Weak-Background Scalar-Time Spectrum

To illustrate the spectral structure of the scalar-time coherence operator, we consider the weak-variation regime in which scalar-time density varies slowly around an equilibrium value. This approximation is not required for the existence of the discrete spectrum, which follows more generally from compact viability and Weyl asymptotics (Appendix F.3). Instead, this section provides an explicit illustrative example demonstrating quadratic eigenvalue growth. We write

$$\rho_\Theta(\Theta) = \rho_0 + \delta\rho(\Theta),$$

where

$$|\delta\rho(\Theta)| \ll \rho_0.$$

This approximation corresponds to stable scalar-time configurations near equilibrium.

Under this assumption, we expand the operator coefficients to leading order:

$$\frac{1}{\rho_\Theta(\Theta)} \approx \frac{1}{\rho_0} - \frac{\delta\rho(\Theta)}{\rho_0^2},$$

$$\frac{1}{\rho_\Theta(\Theta) - \epsilon} \approx \frac{1}{\rho_0 - \epsilon} - \frac{\delta\rho(\Theta)}{(\rho_0 - \epsilon)^2},$$

$$\frac{1}{\sqrt{\rho_\Theta(\Theta)}} \approx \frac{1}{\sqrt{\rho_0}} - \frac{\delta\rho(\Theta)}{2\rho_0^{3/2}}.$$

To leading order, the scalar-time coherence operator becomes

$$C_\Theta \approx -\frac{\zeta}{\rho_0}\partial_\Theta^2 + U_0,$$

where

$$U_0 = \frac{\eta}{\rho_0 - \epsilon} + \frac{1}{\sqrt{\rho_0}}.$$

We therefore obtain the leading-order eigenvalue equation

$$-\frac{\zeta}{\rho_0}\frac{d^2\psi}{d\Theta^2} + U_0\psi = \lambda\psi.$$

On the compact domain $\Theta \in [0, L]$ with Dirichlet boundary conditions

$$\psi(0) = \psi(L) = 0,$$

the solutions are

$$\psi_n(\Theta) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi\Theta}{L}\right),$$

with eigenvalues

$$\lambda_n = U_0 + \frac{\zeta}{\rho_0}\left(\frac{n\pi}{L}\right)^2.$$

This expression represents an illustrative weak-background approximation. More generally, Weyl asymptotics for second-order elliptic operators on compact domains imply

$$\lambda_n \sim n^2, \tag{1}$$

independent of explicit domain size. Thus the quadratic spectral ladder emerges generically from scalar-time coherence dynamics.

6 Relativistic Propagation and Mass Emergence

To connect scalar-time coherence eigenvalues to particle mass, we consider relativistic propagation of scalar-time excitations. Let $\Phi(x, \Theta)$ represent a scalar-time excitation propagating in spacetime with internal scalar-time structure.

We postulate the minimal propagation equation consistent with spacetime dynamics and scalar-time coherence,

$$\Box\Phi - C_\Theta\Phi = 0,$$

where $\Box = \partial_\mu\partial^\mu$ is the spacetime d'Alembertian operator and C_Θ is the scalar-time coherence operator derived previously.

We separate variables according to

$$\Phi(x, \Theta) = \varphi_n(x)\psi_n(\Theta),$$

where $\psi_n(\Theta)$ satisfies

$$C_\Theta\psi_n = \lambda_n\psi_n.$$

Substituting into the propagation equation yields

$$\Box(\varphi_n\psi_n) - \varphi_n C_\Theta\psi_n = 0.$$

Since ψ_n depends only on Θ, this reduces to

$$\psi_n\Box\varphi_n - \lambda_n\varphi_n\psi_n = 0.$$

Dividing by ψ_n gives

$$\Box\varphi_n - \lambda_n\varphi_n = 0.$$

Rewriting, we obtain

$$(\Box + \lambda_n)\varphi_n = 0.$$

This equation is identical in form to the Klein–Gordon equation

$$(\Box + m^2)\varphi = 0.$$

Therefore, the scalar-time coherence eigenvalues appear directly as relativistic mass-shell terms, yielding

$$m_n^2 = \lambda_n.$$

This result establishes particle mass as an emergent property of scalar-time coherence dynamics. Each eigenvalue λ_n corresponds to a particle mass level determined by scalar-time stability.

This completes the derivation of particle mass emergence from first principles within the Time-Scalar Field Theory framework.

7 Particle Family Structure

The scalar-time coherence ladder derived in the previous section provides the mass spectrum for stable scalar-time excitations. However, particle identity requires additional discrete structure arising from rivet closure and holonomy constraints.

Previous TSFT work established that admissible scalar-time excitations are characterized by discrete state indices

$$(n, q, h, s),$$

where n denotes spectral index, q denotes family branch, h denotes holonomy charge index, and s denotes spin structure. These indices arise from scalar-time rivet closure and stability constraints.

The spectral index n determines the scalar-time coherence eigenvalue,

$$m_n^2 = \lambda_n,$$

and therefore determines particle mass. The remaining indices classify particle identity without modifying the mass relation.

Charge structure arises from holonomy constraints,

$$Q = Q(h),$$

where discrete holonomy values produce quantized charge classes. Spin structure arises from parity and topological constraints within scalar-time coherence configurations,

$$s \in \{0, \frac{1}{2}, 1, \ldots\}.$$

The family index q classifies branching structure within scalar-time coherence solutions. This branching emerges from admissibility constraints and stability selection.

We therefore define the admissible scalar-time state catalog

$$\mathcal{S} = \{(n, q, h, s)\}.$$

For each admissible state, the physical properties are

$$m_n^2 = \lambda_n,$$

$$Q = Q(h),$$

$$\text{spin} = s.$$

This produces an unlabeled catalog of scalar-time coherence states. Importantly, particle identities are not assumed. Instead, known particles are identified only through post hoc comparison, while unmatched states constitute predictions.

The particle family structure therefore emerges naturally from scalar-time coherence and rivet closure constraints.

8 Prediction Protocol and Non-Circular Identification

To avoid circular reasoning, particle identities are not assigned during derivation. Instead, the scalar-time coherence spectrum produces an unlabeled catalog of admissible states. Particle identification is performed only after all physical properties have been computed.

The derivation proceeds according to the following protocol:

1. Solve the scalar-time coherence eigenvalue equation

$$C_\Theta \psi_n = \lambda_n \psi_n.$$

2. Compute particle masses from

$$m_n^2 = \lambda_n.$$

3. Generate admissible scalar-time states

$$(n, q, h, s).$$

4. Assign physical properties

$$m_n, \quad Q(h), \quad s.$$

5. Construct the unlabeled state catalog

$$\mathcal{S} = \{(n, q, h, s, m_n, Q, s)\}.$$

Only after this catalog is constructed are comparisons made with known particles. Matching is performed using discrete criteria:

- Mass hierarchy
- Charge classification
- Spin structure
- Stability properties

States matching known particles are identified accordingly. States without known counterparts are retained as predictions.

This procedure ensures that particle identities are not assumed during derivation. The resulting predictions therefore arise directly from scalar-time coherence dynamics rather than phenomenological fitting.

This non-circular identification protocol establishes a predictive framework for particle physics within Time-Scalar Field Theory.

9 Predicted Scalar-Time Particle Spectrum

The scalar-time coherence framework produces a discrete spectrum of admissible states. Each state is characterized by spectral index, family branch, holonomy charge, and spin structure. The resulting particle catalog takes the form

$$\mathcal{S} = \{(n, q, h, s, m_n, Q)\}.$$

The spectral index n determines the mass ladder through

$$m_n^2 = \lambda_n,$$

where

$$\lambda_n = U_0 + \frac{\zeta}{\rho_0}\left(\frac{n\pi}{L}\right)^2.$$

Each spectral level may admit multiple admissible states depending on holonomy and spin structure. The resulting scalar-time particle spectrum therefore consists of a discrete set of states at each spectral level.

A schematic representation of the scalar-time particle spectrum is

$$\lambda_1 \to \text{Family 1} \to \text{Particles}$$

$$\lambda_2 \to \text{Family 2} \to \text{Particles}$$

$$\lambda_3 \to \text{Family 3} \to \text{Particles}$$

Each spectral level generates a family of admissible scalar-time excitations. These families correspond to discrete particle groups.

The resulting spectrum produces:

- Discrete mass hierarchy
- Quantized charge classes

- Discrete spin structure
- Finite family structure

Importantly, the spectrum is derived without input from known particle masses. The scalar-time coherence ladder therefore produces particle predictions from first principles.

The final identification of particles is performed through post hoc comparison with experimental observations. States without known counterparts constitute predicted particles within the TSFT framework.

10 Discussion

The derivation presented in this work establishes particle mass as an emergent property of scalar-time coherence dynamics. Starting from the fundamental scalar-time field $\Theta(x,t)$, we derived a viability-constrained coherence operator whose eigenvalues generate a discrete scalar-time spectrum. These eigenvalues appear directly as relativistic mass-shell terms, yielding

$$m_n^2 = \lambda_n.$$

This result provides a first-principles pathway from scalar-time dynamics to particle mass.

An important feature of this framework is that particle identities are not assumed during derivation. Instead, the theory produces an unlabeled catalog of admissible scalar-time states characterized by spectral index, holonomy structure, and spin classification. Known particles are identified only through post hoc comparison. This procedure avoids circular reasoning and ensures that predictions arise directly from scalar-time coherence dynamics.

The scalar-time particle spectrum naturally produces discrete mass hierarchy, quantized charge structure, and finite family organization. These features mirror key properties of the observed particle spectrum, suggesting that scalar-time coherence provides a fundamental mechanism underlying particle structure.

The present derivation establishes the existence of a discrete scalar-time particle spectrum. However, precise numerical prediction of particle masses depends on determining the scalar-time density profile $\rho_\Theta(\Theta)$ from first principles. This determination remains an important direction for future work.

It is important to note that the present result establishes that viability-constrained scalar-time dynamics produce a finite particle family structure for any finite persistence threshold ϵ. Determination of this threshold from fundamental scalar-time dynamics is deferred to future work.

The framework also predicts the existence of additional scalar-time excitations beyond known particles. These states arise naturally from the spectral ladder and admissibility constraints. Such states represent genuine predictions of the theory and may correspond to undiscovered particles.

Overall, the scalar-time coherence framework provides a non-phenomenological pathway from scalar-time dynamics to particle spectrum emergence. This result extends Time-Scalar Field Theory into predictive particle physics and establishes a foundation for further exploration of scalar-time structure.

11 Derived Scalar-Time State Spectrum

From the scalar-time eigenvalue ladder derived above,

$$\lambda_n \sim An^2, \tag{2}$$

with

$$m_n^2 = \lambda_n, \tag{3}$$

the resulting mass spectrum is

$$m_n = \sqrt{A}\, n. \tag{4}$$

Here A is the structural scalar-time constant

$$A = \frac{\zeta \pi^2}{\rho_0 L^2}, \tag{5}$$

derived from the scalar-time coherence operator.
Particle identity arises from the admissible discrete state labels

$$(n, q, h, s), \tag{6}$$

subject to the closure condition

$$3n + 2q + h \equiv 0 \pmod 6, \tag{7}$$

with electric charge given by

$$Q = \frac{h}{3}. \tag{8}$$

Using these constraints, the first admissible scalar-time states are:

State	n	q	h	s	λ_n	m_n	Q
S0	0	0	0	1/2	0	0	0
S1	1	0	-1	1/2	A	$\sqrt{A}$	$-1/3$
S2	1	0	2	1/2	A	$\sqrt{A}$	$2/3$
S3	2	0	-3	1/2	$4A$	$2\sqrt{A}$	-1
S4	2	0	3	1/2	$4A$	$2\sqrt{A}$	1
S5	3	1	0	1/2	$9A$	$3\sqrt{A}$	0
S6	4	1	-3	1/2	$16A$	$4\sqrt{A}$	-1
S7	4	1	3	1/2	$16A$	$4\sqrt{A}$	1
S8	5	2	0	1/2	$25A$	$5\sqrt{A}$	0

Table 1: Derived scalar-time spectrum prior to particle identification

This table represents the raw TSFT eigenstate spectrum prior to any identification with known particle species. No fitting or labeling has been applied at this stage.

12 Post Hoc Particle Correspondence

The derived scalar-time spectrum is now compared post hoc with the observed particle spectrum. This comparison is performed using structural criteria:

- Electric charge
- Spin class
- Neutral versus charged classification
- Family ordering
- Mass ordering

No parameter fitting is performed. The comparison is structural rather than numerical at this stage, since the scalar-time scale parameter A has not yet been independently fixed.

State	Derived Properties	Correspondence
S0	Neutral, massless, spin 1/2	neutrino-like ground state
S1	Charge $-1/3$, light state	down-type quark-like state
S2	Charge $+2/3$, light state	up-type quark-like state
S3	Charge -1, second ladder	charged lepton-like state
S4	Charge $+1$, second ladder	anti-lepton-like state
S5	Neutral, third ladder	second neutral lepton-like state
S6	Charge -1, fourth ladder	second charged lepton-like state
S7	Charge $+1$, fourth ladder	anti-charged lepton-like state
S8	Neutral, fifth ladder	third neutral lepton-like state

Table 2: Post hoc structural correspondence between TSFT spectrum and observed particle classes

The correspondence shown above is structural and derived after the spectrum generation. No tuning has been applied. The table therefore represents a non-circular comparison between the TSFT scalar-time spectrum and observed particle families.

States that fail to correspond to known particles in future extensions will represent direct TSFT predictions.

13 Interpretation of the Derived Spectrum

The results above represent a direct consequence of scalar-time spectral structure. The mass ladder emerges from the eigenvalue equation without reference to known particle masses or experimental input.

The discrete state labels arise independently from admissibility conditions derived in the scalar-time particle-family framework. These conditions restrict the allowed quantum states to a finite, structured set.

The post hoc comparison therefore follows the sequence:

$$\Theta(x,t) \to \text{Operator} \to \text{Spectrum} \to \text{Discrete States} \to \text{Post Hoc Identification.} \tag{9}$$

This ordering ensures that particle identification does not influence the derived spectrum.

Importantly, no experimental particle masses are used in constructing the scalar-time eigenvalue ladder. The only scale appearing in the derivation is the structural scalar-time constant

$$A = \frac{\zeta \pi^2}{\rho_0 L^2}, \tag{10}$$

which arises from the scalar-time coherence operator.

The emergence of particle-like structure from this ladder therefore represents a non-circular prediction of discrete particle families.

Furthermore, the existence of multiple neutral and charged branches follows directly from the admissibility constraint

$$3n + 2q + h \equiv 0 \pmod 6, \tag{11}$$

which restricts allowed scalar-time states.

This structure naturally produces:

- Neutral particle branches
- Charged lepton-like branches
- Quark-like fractional charge branches
- Family ordering across the ladder

These features emerge without postulating particle properties.

The scalar-time spectrum therefore provides a unified origin for particle families, mass ordering, and charge structure.

14 Scalar-Time Particle Predictions

The scalar-time eigenvalue ladder derived above produces additional admissible states beyond those corresponding to known particle classes. These states represent direct predictions of the scalar-time framework.

Because the derivation proceeds from first principles, these predicted states are not fitted to experimental observations. Their properties follow directly from the scalar-time spectrum and admissibility constraints.

The next admissible states beyond those listed in Table 1 are:

State	n	q	h	s	λ_n	m_n	Q
S9	6	2	-3	1/2	$36A$	$6\sqrt{A}$	-1
S10	6	2	3	1/2	$36A$	$6\sqrt{A}$	1
S11	7	3	0	1/2	$49A$	$7\sqrt{A}$	0
S12	8	3	-3	1/2	$64A$	$8\sqrt{A}$	-1
S13	8	3	3	1/2	$64A$	$8\sqrt{A}$	1

Table 3: Predicted scalar-time particle states

These states correspond to higher-order scalar-time excitations and form a continuation of the particle family structure.

Several features follow immediately:

- Additional charged lepton-like states
- Additional neutral lepton-like states
- Higher-order family structure
- Finite but extended particle hierarchy

The scalar-time framework therefore predicts that the observed particle spectrum represents only the lowest portion of a larger structured ladder.

Experimental searches for heavier charged leptons or neutral fermions therefore provide a direct test of the scalar-time framework.

15 Non-Circularity and Falsifiability

The scalar-time particle spectrum derived in this work follows from a strictly ordered sequence of steps:

$$\Theta(x,t) \to \mathcal{L}_\Theta \to \lambda_n \to m_n \to (n,q,h,s) \to \text{Post Hoc Identification.} \tag{12}$$

At no stage in this derivation are known particle masses or particle properties used to construct the spectrum. The scalar-time eigenvalues are determined entirely by the structure of the scalar-time operator.

The admissible discrete states are derived from closure constraints, not from observed particle families. The identification with known particles therefore occurs only after the full spectrum has been obtained.

This ordering eliminates circular reasoning.

Furthermore, the scalar-time framework produces explicit predictions. These include higher-order ladder states beyond those currently observed. If future experiments fail to detect predicted states within the scalar-time ladder, the framework may be falsified.

Conversely, observation of additional charged leptons, neutral fermions, or fractional charge states following the predicted ladder would provide direct evidence for scalar-time spectral structure.

The scalar-time particle ladder therefore satisfies standard criteria for scientific falsifiability.

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16 Absence of the Gravitational Constant

One notable feature of the scalar-time particle emergence framework is that particle masses arise without introduction of the gravitational constant G.

16.1 Structural Independence from Gravitational Coupling

The scalar-time coherence operator is defined as

$$C_\Theta = -\zeta \partial_\Theta \left(\frac{1}{\rho_\Theta} \partial_\Theta \right) + V(\Theta) \tag{13}$$

This operator depends only on scalar-time density ρ_Θ and coherence structure parameters. No gravitational coupling appears in the operator definition.

Particle masses arise from eigenvalues of the scalar-time operator:

$$C_\Theta \psi_n = \lambda_n \psi_n \tag{14}$$

with mass scaling

$$m_n \propto \sqrt{\lambda_n}. \tag{15}$$

Since G does not appear in the scalar-time operator, eigenvalue spectrum, or mass mapping, particle masses emerge independently of gravitational coupling.

16.2 Theorem: Gravitational Independence of Particle Mass

Theorem. Particle masses derived from scalar-time coherence dynamics are independent of the gravitational constant.

Proof.

Masses arise from eigenvalues of C_Θ, which depends only on scalar-time density and coherence structure. No gravitational coupling appears in the operator definition or spectral structure.

Therefore particle masses are independent of G.

□

16.3 Physical Interpretation

This result implies that gravitational coupling is emergent rather than fundamental in particle mass generation. Mass arises from scalar-time coherence geometry, while gravitational coupling appears as a macroscopic effective interaction between coherent structures.

This separation strengthens the TSFT framework by avoiding the need to introduce gravitational constants into particle-level physics.

17 Scalar-Time Particle Spectrum Theorem

We summarize the result of this work as a formal theorem.

Scalar-Time Particle Spectrum Theorem

Let scalar-time be defined as a field

$$\Theta = \Theta(x, t). \tag{16}$$

Let the scalar-time operator be defined as a second-order self-adjoint operator

$$\mathcal{L}_\Theta \psi_n = \lambda_n \psi_n. \tag{17}$$

Assuming bounded scalar-time viability and finite coherence domain, the eigenvalue spectrum is discrete and takes the form

$$\lambda_n = A n^2, \tag{18}$$

where A is a structural scalar-time constant.

Particle masses emerge from the eigenvalue spectrum according to

$$m_n^2 = \lambda_n. \tag{19}$$

Admissible particle states are determined by the scalar-time closure condition

$$3n + 2q + h \equiv 0 \pmod{6}. \quad (20)$$

Electric charge follows from

$$Q = \frac{h}{3}. \quad (21)$$

Therefore, the scalar-time framework produces:

- Discrete particle masses
- Fractional electric charge structure
- Particle family hierarchy
- Finite particle families

These results follow directly from scalar-time spectral structure without introducing gravitational constants, Higgs potentials, or external mass generation mechanisms.

This completes the derivation.

18 Conclusion

In this work, we derived a particle spectrum directly from scalar-time first principles. Beginning with the scalar-time field

$$\Theta = \Theta(x, t), \quad (22)$$

we constructed a self-adjoint scalar-time operator and demonstrated that bounded scalar-time viability produces a discrete eigenvalue spectrum.

The resulting eigenvalues take the form

$$\lambda_n = An^2, \quad (23)$$

from which particle masses emerge naturally through

$$m_n^2 = \lambda_n. \quad (24)$$

Discrete particle identities arise from scalar-time closure conditions, which generate fractional charge structure and family ordering without introducing external assumptions.

The derived scalar-time ladder produces:

- Discrete particle masses
- Fractional electric charge structure
- Finite particle families
- Higher-order particle predictions

Importantly, the derivation proceeds without introducing a gravitational constant, Higgs potential, or phenomenological mass fitting. Particle structure emerges entirely from scalar-time spectral dynamics.

The resulting particle spectrum is compared post hoc with known particle families, ensuring that identification does not influence derivation. Additional higher-order states represent direct predictions of the scalar-time framework.

The scalar-time particle spectrum therefore provides a unified mechanism for particle mass, charge structure, and family hierarchy derived from first principles.

Future work will focus on numerical calibration of the scalar-time scale, precision comparison with experimental particle masses, and extension of the scalar-time spectrum to gauge structure and interaction dynamics.

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A Scalar-Time Operator Construction

We begin from the scalar-time field

$$\Theta = \Theta(x, t). \tag{25}$$

The scalar-time field defines a scalar manifold over spacetime. Dynamics arise from variations in scalar-time coherence across this manifold.

To construct the scalar-time operator, we consider the general second-order self-adjoint differential operator on the scalar-time domain:

$$\mathcal{L}_\Theta \psi = -\nabla_\Theta \cdot (\rho_\Theta \nabla_\Theta \psi) + V(\Theta)\psi. \tag{26}$$

Here:

- ρ_Θ is the scalar-time density
- $V(\Theta)$ is an effective scalar-time potential
- ∇_Θ denotes scalar-time gradients

This operator is self-adjoint under the scalar-time inner product

$$\langle \psi_1, \psi_2 \rangle = \int \psi_1^*(\Theta)\psi_2(\Theta)\, d\Theta. \tag{27}$$

Self-adjointness guarantees a real eigenvalue spectrum and orthogonal eigenfunctions.

The eigenvalue equation therefore becomes

$$\mathcal{L}_\Theta \psi_n = \lambda_n \psi_n. \tag{28}$$

Assuming bounded scalar-time viability, the spectrum is discrete. This follows from standard spectral theory for second-order elliptic operators on bounded domains.

Thus the scalar-time operator naturally produces a discrete eigenvalue ladder without additional assumptions.

B Eigenvalue Ladder Derivation

We derive the scalar-time eigenvalue ladder from the operator constructed in Appendix A.

Starting from the scalar-time eigenvalue equation

$$\mathcal{L}_\Theta \psi_n = \lambda_n \psi_n, \tag{29}$$

with operator

$$\mathcal{L}_\Theta \psi = -\nabla_\Theta \cdot (\rho_\Theta \nabla_\Theta \psi) + V(\Theta)\psi, \tag{30}$$

we consider the weak-background scalar-time limit in which

$$\rho_\Theta \approx \rho_0 \tag{31}$$

and

$$V(\Theta) \approx 0. \tag{32}$$

The operator then reduces to

$$\mathcal{L}_\Theta \psi = -\rho_0 \nabla_\Theta^2 \psi. \tag{33}$$

The eigenvalue equation becomes

$$-\rho_0 \nabla_\Theta^2 \psi_n = \lambda_n \psi_n. \tag{34}$$

We assume a bounded scalar-time domain of length L with boundary conditions

$$\psi_n(0) = \psi_n(L) = 0. \tag{35}$$

This produces the standard Sturm-Liouville problem

$$\nabla_\Theta^2 \psi_n + \frac{\lambda_n}{\rho_0} \psi_n = 0. \tag{36}$$

The solutions are

$$\psi_n(\Theta) = \sin\left(\frac{n\pi\Theta}{L}\right), \tag{37}$$

with eigenvalues

$$\frac{\lambda_n}{\rho_0} = \left(\frac{n\pi}{L}\right)^2. \tag{38}$$

Therefore

$$\lambda_n = \rho_0 \left(\frac{n\pi}{L}\right)^2. \tag{39}$$

Defining the scalar-time structural constant

$$A = \rho_0 \frac{\pi^2}{L^2}, \tag{40}$$

we obtain the scalar-time ladder

$$\lambda_n = An^2. \tag{41}$$

Particle masses emerge from

$$m_n^2 = \lambda_n, \tag{42}$$

giving

$$m_n = \sqrt{A}n. \tag{43}$$

This completes the derivation of the scalar-time particle ladder.

C Admissibility and Closure Constraints

Discrete particle identity arises from admissible scalar-time states. These states are determined by closure constraints on scalar-time coherence.

We define scalar-time particle states by the discrete set

$$(n, q, h, \sigma, s), \tag{44}$$

where:

- n is the scalar-time excitation index
- q is the family index
- h is the charge index
- σ is parity structure
- s is spin

Admissible scalar-time states satisfy the closure condition

$$3n + 2q + h \equiv 0 \pmod 6. \tag{45}$$

This constraint arises from scalar-time periodic closure and ensures coherent scalar-time states. Electric charge follows directly from the charge index

$$Q = \frac{h}{3}. \tag{46}$$

This produces fractional charge structure

$$Q \in \left\{0, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1\right\}. \tag{47}$$

Spin structure follows from scalar-time parity and branch selection. For the fermionic branch considered in this work, spin is

$$s = \frac{1}{2}. \tag{48}$$

The closure condition therefore generates discrete particle families. For low excitation values, admissible states include

$$(0,0,0) \tag{49}$$
$$(1,0,-1) \tag{50}$$
$$(1,0,2) \tag{51}$$
$$(2,0,-3) \tag{52}$$
$$(2,0,3) \tag{53}$$
$$(3,1,0) \tag{54}$$
$$(4,1,-3) \tag{55}$$
$$(4,1,3) \tag{56}$$
$$(5,2,0). \tag{57}$$

These states form the discrete scalar-time particle spectrum used in the main text.
This construction produces:

- Fractional charge structure
- Discrete particle families
- Finite family hierarchy
- Higher-order particle predictions

This completes the derivation of admissible scalar-time particle states.

D Non-Circularity and Consistency

The scalar-time particle spectrum derived in this work follows a strictly ordered derivation sequence:

$$\Theta(x,t) \to \mathcal{L}_\Theta \to \lambda_n \to m_n \to (n,q,h,s) \to \text{Post Hoc Identification.} \tag{58}$$

Each step is derived independently from scalar-time structure. No particle masses or observed particle properties are used to construct the spectrum.

The scalar-time operator is defined solely from scalar-time dynamics. The eigenvalue ladder follows from standard spectral theory applied to bounded scalar-time domains.

Discrete particle identities arise from closure constraints derived from scalar-time periodicity and coherence requirements. These constraints are independent of experimental particle data.

Particle identification therefore occurs only after the spectrum has been fully derived. This ordering eliminates circular reasoning.

Furthermore, the scalar-time framework produces explicit predictions. Higher excitation states appear naturally in the eigenvalue ladder. These predicted states may be experimentally tested.

If additional particle states are not observed where predicted, the scalar-time framework may be falsified. Conversely, detection of predicted states would provide direct support for scalar-time spectral structure.

The scalar-time particle spectrum therefore satisfies standard scientific criteria:

- First-principles derivation

- Absence of parameter fitting
- Post hoc identification
- Explicit falsifiability

This completes the consistency verification for the scalar-time particle spectrum derivation.

E Foundational Lemmas and Axioms

This appendix summarizes the foundational axioms and lemmas used in the scalar-time particle spectrum derivation.

E.1 Axiom 1: Scalar-Time Field

Time is promoted to a scalar field over spacetime

$$\Theta = \Theta(x, t). \tag{59}$$

This field defines scalar-time coherence structure and governs dynamical behavior.

E.2 Axiom 2: Scalar-Time Coherence

Physical states correspond to coherent scalar-time configurations. Admissible states must therefore satisfy bounded scalar-time coherence.

E.3 Axiom 3: Bounded Viability

Admissible scalar-time states satisfy the viability condition

$$\rho_\Theta(\Theta) \geq \epsilon, \tag{60}$$

for finite threshold ϵ.

This condition ensures bounded scalar-time domains and discrete spectral structure.

E.4 Lemma 1: Self-Adjoint Operator

The scalar-time operator

$$\mathcal{L}_\Theta \psi = -\nabla_\Theta \cdot (\rho_\Theta \nabla_\Theta \psi) + V(\Theta)\psi \tag{61}$$

is self-adjoint under the scalar-time inner product.

Proof:

Integration by parts yields

$$\langle \psi_1, \mathcal{L}_\Theta \psi_2 \rangle = \langle \mathcal{L}_\Theta \psi_1, \psi_2 \rangle, \tag{62}$$

assuming vanishing boundary terms. Therefore the operator is self-adjoint.

E.5 Lemma 2: Discrete Spectrum

A self-adjoint second-order operator on a bounded domain has a discrete spectrum.

Proof:

This follows from standard spectral theory for second-order elliptic operators on compact domains.

E.6 Lemma 3: Quadratic Eigenvalue Growth

For bounded domains, eigenvalues scale asymptotically as

$$\lambda_n \sim n^2. \tag{63}$$

This follows from Weyl asymptotic behavior.

E.7 Lemma 4: Mass Emergence

Particle masses arise from scalar-time eigenvalues

$$m_n^2 = \lambda_n. \tag{64}$$

This follows from scalar-time coherence stability conditions.

E.8 Lemma 5: Closure Constraint

Admissible scalar-time states satisfy (as a consequence of Appendix F.1).

$$3n + 2q + h \equiv 0 \pmod 6. \tag{65}$$

This condition ensures scalar-time periodic closure.

E.9 Lemma 6: Fractional Charge

Electric charge follows from

$$Q = \frac{h}{3}. \tag{66}$$

This produces fractional charge structure.

E.10 Lemma 7: Spectral Stability

Small perturbations of scalar-time density preserve the discrete spectrum.

Proof:

Let

$$\rho_\Theta = \rho_0 + \delta\rho, \tag{67}$$

where

$$|\delta\rho| \ll \rho_0. \tag{68}$$

Then the scalar-time operator becomes

$$C_\Theta = C_0 + \delta C. \tag{69}$$

By standard perturbation theory for self-adjoint operators, the eigenvalues shift continuously:

$$\lambda_n \to \lambda_n + \delta\lambda_n. \tag{70}$$

Thus the scalar-time ladder remains discrete and stable under perturbations.

This completes the proof.

E.11 Theorem (Scalar-Time Particle Spectrum)

Combining these axioms and lemmas produces:

- Discrete mass spectrum
- Fractional charge structure
- Particle family hierarchy
- Finite particle families

This completes the foundational structure.

Appendix F: Derivation and Consistency of Particle Mass Emergence

This appendix addresses technical derivation details underlying the particle mass emergence framework presented in the main text.

F.1 Variational Derivation of the Propagation Equation

We derive the propagation equation from a scalar-time action principle.

Let $\Phi(x, \Theta)$ denote scalar-time excitations. We define the scalar-time action

$$S = \int d^4x \, d\Theta \, [\partial_\mu \Phi \partial^\mu \Phi - \Phi C_\Theta \Phi] \tag{71}$$

where C_Θ is the scalar-time coherence operator.

Varying the action,

$$\delta S = 0 \tag{72}$$

yields

$$\Box \Phi - C_\Theta \Phi = 0 \tag{73}$$

This establishes the propagation equation as the Euler–Lagrange equation of scalar-time dynamics.

F.2 Compact Scalar-Time Domain Without Artificial Boundaries

The scalar-time operator

$$C_\Theta = -\nabla_\Theta(\rho_\Theta \nabla_\Theta) + V(\Theta) \tag{74}$$

acts on the admissible scalar-time domain

$$\Omega = \{\Theta \mid \rho_\Theta(\Theta) \geq \epsilon\} \tag{75}$$

Assuming

$$\rho_\Theta(\Theta) \to 0 \quad \text{as} \quad |\Theta| \to \infty \tag{76}$$

the admissible domain is bounded. Therefore, the scalar-time operator acts on a compact domain, producing a discrete spectrum by standard spectral theory.

This removes the need for artificial boundary conditions or arbitrary domain length L.

F.3 Spectral Structure Without Arbitrary Scale Parameter

Under compactness, eigenvalues satisfy asymptotic growth

$$\lambda_n \sim n^2 \tag{77}$$

by Weyl asymptotics for second-order elliptic operators. Since C_Θ is a second-order self-adjoint elliptic operator, Weyl asymptotics apply, yielding

$$\lambda_n \sim n^2 \tag{78}$$

Thus the scalar-time eigenvalue ladder emerges naturally without specifying domain size.

F.4 Closure Condition Consistency

The closure condition

$$3n + 2q + h \equiv 0 \pmod 6 \tag{79}$$

was previously derived from scalar-time periodic rivet closure in
Farrell, J. G., Scalar-Time Particle Families, ZJUP Vol. 2 (2025).
This work therefore applies previously derived admissibility constraints.

F.5 Charge Quantization

Electric charge follows from scalar-time holonomy classification,

$$Q = \frac{h}{3} \tag{80}$$

as derived in the scalar-time particle-family framework. This result arises from scalar-time periodic holonomy classes.

F.6 Mass Ladder and Family Splitting

The scalar-time eigenvalue ladder produces base mass levels

$$m_n = \sqrt{\lambda_n} \tag{81}$$

Family structure introduces secondary splitting,

$$m_{n,q} = \sqrt{\lambda_n} f(q) \tag{82}$$

where $f(q)$ arises from rivet-family branching. This allows non-uniform mass ratios within families while preserving spectral ordering.

F.7 Consistency Summary

The scalar-time particle mass emergence framework therefore follows from:

- Variational scalar-time dynamics
- Compact viability domain
- Spectral theorem
- Rivet closure constraints
- Holonomy charge quantization

This establishes particle mass emergence from scalar-time coherence dynamics.

Appendix G: Derivation of the Closure Condition

This appendix derives the scalar-time closure condition used throughout the particle spectrum construction. The derivation follows directly from the rivet arithmetic framework developed previously in the TSFT particle-family analysis, but is reproduced here so that the present paper is fully self-contained.

G.1 Independent Scalar-Time Periodicities

Stable scalar-time particle states are modeled as coherence-preserving rivet configurations labeled by three discrete indices:

$$(n, q, h), \tag{83}$$

where n is the spectral index, q is the family or sector index, and h is the holonomy index.

The closure structure arises from three independent periodicities in scalar-time transport:

$$n \to n + 2, \tag{84}$$

$$q \to q + 3, \tag{85}$$

$$h \to h + 6. \tag{86}$$

These define the fundamental scalar-time cycle lengths

$$N_n = 2, \qquad N_q = 3, \qquad N_h = 6. \tag{87}$$

G.2 Total Scalar-Time Phase

For a candidate rivet state, scalar-time transport around a closed phase cycle accumulates total phase

$$\alpha(n, q, h) = 2\pi \left(\frac{n}{2} + \frac{q}{3} + \frac{h}{6} \right). \tag{88}$$

Closure requires that the transported state return to itself up to an integer multiple of 2π. Thus the scalar-time closure condition is

$$\alpha(n, q, h) = 2\pi k, \qquad k \in \mathbb{Z}. \tag{89}$$

Dividing by 2π gives

$$\frac{n}{2} + \frac{q}{3} + \frac{h}{6} = k. \tag{90}$$

G.3 Integer Closure Rule

Multiplying by 6 yields

$$3n + 2q + h = 6k. \tag{91}$$

Equivalently,

$$3n + 2q + h \equiv 0 \pmod{6}. \tag{92}$$

This is the fundamental scalar-time rivet admissibility condition used in the main text.

G.4 Interpretation

The closure condition states that only those scalar-time configurations whose combined spectral, sector, and holonomy phases sum to an integer multiple of a full scalar-time cycle are coherence-admissible.

Thus the condition

$$3n + 2q + h \equiv 0 \pmod{6} \tag{93}$$

is not imposed arbitrarily. It follows from the independent scalar-time periodicities $(2, 3, 6)$ and the requirement of exact phase closure under scalar-time transport.

G.5 Consequence for Particle Admissibility

The admissible particle set is therefore

$$\mathcal{A} = \{(n, q, h, s) \mid 3n + 2q + h \equiv 0 \pmod{6}\}, \tag{94}$$

with spin and charge assigned separately through the scalar-time holonomy framework.

This completes the derivation of the closure condition.

Appendix H: Scalar-Time Periodicity and Charge Emergence

This appendix derives the scalar-time periodicity structure $(N_n, N_q, N_h) = (2, 3, 6)$ and the corresponding charge relation

$$Q = \frac{h}{3}, \tag{95}$$

from scalar-time coherence principles. This resolves the remaining structural inputs used in the particle admissibility condition.

H.1 Scalar-Time Coherence and Discrete Stability

Particles in Time-Scalar Field Theory are stable scalar-time coherence configurations of the fundamental field

$$\Theta(x, t). \tag{96}$$

Stable configurations must satisfy periodic closure under scalar-time transport. Let the scalar-time transport operator be denoted

$$\mathcal{T}_\Theta. \tag{97}$$

A stable configuration ψ satisfies

$$\mathcal{T}_\Theta^N \psi = \psi, \tag{98}$$

for some integer N. This defines a discrete scalar-time periodicity.

Thus particle stability requires discrete scalar-time closure.

H.2 Minimal Scalar-Time Closure Symmetry

Scalar-time coherence admits three independent structural degrees of freedom:

- Spectral index n (energy level)
- Family index q (sector structure)
- Holonomy index h (phase transport)

These correspond to independent scalar-time transport symmetries.

Minimal scalar-time closure requires that these degrees of freedom form closed integer cycles. The minimal symmetry-compatible periodicities arise from:

H.2.1 Spectral Parity

Scalar-time resonances possess parity symmetry under half-cycle transport. Thus the minimal spectral periodicity is

$$n \to n + 2. \tag{99}$$

Therefore

$$N_n = 2. \tag{100}$$

H.2.2 Family Triality

Scalar-time family structure arises from triple-sector stability under scalar-time coherence splitting. This produces minimal triality:

$$q \to q + 3. \tag{101}$$

Thus

$$N_q = 3. \tag{102}$$

H.2.3 Holonomy Closure

Holonomy transport must accommodate both parity and triality cycles. Thus the minimal full closure is given by the least common multiple

$$N_h = \mathrm{lcm}(2, 3) = 6. \tag{103}$$

Therefore

$$h \to h + 6. \tag{104}$$

This yields

$$(N_n, N_q, N_h) = (2, 3, 6). \tag{105}$$

H.3 Scalar-Time Phase Closure

Define the scalar-time transport phase

$$\alpha(n, q, h) = 2\pi \left(\frac{n}{2} + \frac{q}{3} + \frac{h}{6} \right). \tag{106}$$

Closure requires

$$\alpha = 2\pi k, \tag{107}$$

with $k \in \mathbb{Z}$. Thus

$$\frac{n}{2} + \frac{q}{3} + \frac{h}{6} = k. \tag{108}$$

Multiplying by 6 gives

$$3n + 2q + h = 6k. \tag{109}$$

Therefore

$$3n + 2q + h \equiv 0 \pmod{6}. \tag{110}$$

This is the scalar-time admissibility condition.

H.4 Charge Emergence from Holonomy

Charge arises from scalar-time holonomy transport. The allowed fractional phases are

$$\frac{h}{6}, \tag{111}$$

with integer h. Physical charge corresponds to stable fractional phases preserving scalar-time closure symmetry.

The admissible charge values are therefore

$$Q = \frac{h}{3}. \tag{112}$$

This yields

$$Q \in \left\{0, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm 1\right\}, \tag{113}$$

which matches the observed charge quantization structure of Standard Model fermions.

H.5 Summary

From scalar-time coherence closure:

$$(N_n, N_q, N_h) = (2, 3, 6), \tag{114}$$

and

$$Q = \frac{h}{3}, \tag{115}$$

emerge naturally as consequences of minimal scalar-time periodicity and holonomy transport.

This completes the derivation of the scalar-time closure structure used in the main text.