Logarithmic Closure Flow and the Origin of the Fine-Structure Constant in Time–Scalar Field Theory

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The fine-structure constant $\alpha \approx 1/137$ governs the strength of electromagnetic interaction and appears throughout quantum electrodynamics, atomic spectroscopy, and particle physics. Despite its central importance, the numerical value of α is treated as an empirical input within the Standard Model.

This paper proposes a structural origin for α within the framework of Time–Scalar Field Theory (TSFT). Instead of introducing α as a fundamental coupling constant, we interpret it as a closure residue emerging from repeated scalar-time projection. Physical observables are required to remain stable under cyclic dimensional projection, which imposes a discrete spectral closure condition on the scalar-time evolution operator.

We show that stable closure requires the eigenvalues of the projection operator to lie on the unit circle, producing a quantized phase condition. When scale dependence is introduced through logarithmic scalar-time flow, the resulting closure phase generates an effective dimensionless coupling. The fine-structure constant then appears as the minimal nontrivial closure residue of this operator.

Within this framework, quantum electrodynamics is recovered as an effective theory operating on top of a deeper scalar-time closure structure. The numerical proximity of the resulting coupling to $\alpha^{-1} \approx 137$ emerges as a natural consequence of minimal stable closure rather than an unexplained empirical parameter.

I. INTRODUCTION

The fine-structure constant

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c} \tag{1}$$

is one of the most important dimensionless parameters in modern physics. It governs electromagnetic interaction strength, determines atomic fine-structure splitting, and appears throughout quantum electrodynamics (QED) perturbation theory.

While QED predicts physical observables with extraordinary precision once α is specified, the theory itself does not explain why the constant takes its observed numerical value

$$\alpha^{-1} \approx 137.035999. \tag{2}$$

This has motivated a long history of attempts to derive α from deeper theoretical principles. Approaches have included group-theoretic constructions, cosmological scaling relations, and renormalization arguments. None have produced a widely accepted derivation.

Time–Scalar Field Theory (TSFT) proposes a different perspective. Instead of treating fundamental constants as primitive parameters, TSFT interprets them as residues of dimensional projection from a higher-dimensional scalar-time structure.

In this framework, physical observables emerge when higher-dimensional scalar invariants are repeatedly projected into a lower-dimensional observable sector. Stability requires that such projections eventually close upon themselves after a finite number of steps. The resulting closure condition constrains the allowable eigenvalues of the projection operator.

The central hypothesis of the present work is therefore the following:

> Dimensionless coupling constants arise as residues of stable scalar-time closure.

We develop this idea by constructing a closure operator describing repeated scalar-time projection and deriving the spectral conditions required for stability. When scale dependence is incorporated through logarithmic scalar flow, the closure condition naturally produces a small dimensionless coupling. The resulting structure provides a candidate explanation for the origin of the fine-structure constant.

The purpose of this paper is not to modify the successful predictions of quantum electrodynamics. Instead, we propose that QED operates as an effective field theory on top of a deeper closure structure that determines the value of its coupling constant.

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II. SCALAR–TIME PROJECTION AND CLOSURE OPERATORS

Time–Scalar Field Theory (TSFT) begins from the premise that observable physical quantities arise through repeated projection of higher-dimensional scalar invariants into an observable sector. Rather than assuming constants as primitive inputs, TSFT treats them as residues of stable projection cycles.

Let $\mathcal{I}$ denote a scalar invariant defined in the parent scalar-time manifold. Projection into the observable sector is represented by a linear operator

$$\mathcal{P} : \mathcal{I} \to \mathcal{I}'. \tag{3}$$

Repeated projection generates a sequence

$$\mathcal{I}_{n+1} = \mathcal{P}\mathcal{I}_n. \tag{4}$$

The nth iterate therefore satisfies

$$\mathcal{I}_n = \mathcal{P}^n\mathcal{I}_0. \tag{5}$$

Physical observables correspond to projection sequences that remain stable under repeated iteration. In TSFT this stability requirement is expressed as a closure condition: after a finite number of projections the invariant must return to its original value up to a phase factor.

Definition 1 (Scalar Closure)**.** *A scalar invariant $\mathcal{I}$ satisfies closure after N projections if*

$$\mathcal{P}^N\mathcal{I} = e^{i\theta}\mathcal{I}. \tag{6}$$

The quantity θ represents the accumulated scalar-time phase during one closure cycle. Stability requires that the magnitude of the eigenvalue remain unity,

$$\left|e^{i\theta}\right| = 1, \tag{7}$$

ensuring that projection does not lead to exponential growth or decay.

Thus stable observables correspond to eigenvectors of the projection operator whose eigenvalues lie on the unit circle. Writing

$$\mathcal{P}\psi = \lambda\psi, \tag{8}$$

closure after N projections requires

$$\lambda^N = e^{i2\pi m}, \tag{9}$$

where m is an integer labeling the closure branch.

This immediately implies that the eigenvalues of the projection operator must take the form

$$\lambda = e^{i\theta}, \tag{10}$$

with the phase satisfying the discrete closure condition

$$N\theta = 2\pi m. \tag{11}$$

Equation (12) establishes the fundamental spectral constraint governing stable scalar-time projection.

Within TSFT, interaction strengths correspond to the residual phase accumulated during one projection cycle. A vanishing phase would produce trivial closure, while large phases would destabilize the projection sequence. Physically relevant interactions therefore correspond to small but nonzero closure residues.

In the following section we incorporate scale dependence into the projection operator and show that the closure phase naturally acquires a logarithmic flow structure. This logarithmic accumulation will allow a dimensionless coupling to emerge from the spectral closure condition.

III. LOGARITHMIC CLOSURE FLOW

The spectral closure condition derived in the previous section establishes that stable scalar-time projection requires

$$N\theta = 2\pi m, \tag{12}$$

where θ is the phase accumulated by a single projection and m is an integer labeling the closure branch.

To connect this closure condition with physical coupling strengths, we must account for the fact that physical systems exist across a hierarchy of scales. In particular, the effective structure of interactions often depends on a characteristic energy or length scale μ.

Repeated projection across scales naturally produces multiplicative evolution. If a physical quantity evolves multiplicatively under scale change,

$$X(\mu) = X(\mu_0)\, r^k, \tag{13}$$

then the accumulated change becomes linear when expressed in logarithmic variables,

$$\ln\left(\frac{X(\mu)}{X(\mu_0)}\right) = k \ln r. \tag{14}$$

This logarithmic behavior appears widely in physical systems, including renormalization-group flow in quantum field theory and scaling relations in statistical mechanics. Within the TSFT framework, it arises from repeated scalar-time projection across a hierarchy of observational scales.

We therefore model the closure phase as a function of scale,

$$\theta(\mu) = \theta_0 + \kappa \ln\left(\frac{\mu}{\mu_0}\right), \tag{15}$$

where θ_0 is the reference closure phase at scale μ_0 and κ characterizes the rate at which scalar-time phase accumulates under scale transformation.

Substituting Eq. (16) into the closure condition yields

$$N\left[\theta_0 + \kappa \ln\left(\frac{\mu}{\mu_0}\right)\right] = 2\pi m. \tag{16}$$

Equation (17) expresses the fundamental constraint governing stable scalar-time projection in the presence of scale dependence.

This relation can be interpreted as defining discrete stability branches in the (N, μ) parameter space. For fixed branch index m, stable projection requires a specific balance between the number of projection steps and the accumulated logarithmic phase.

Importantly, the closure condition naturally generates a small dimensionless residue when the closure phase is much smaller than 2π. In this regime, the ratio

$$\delta \equiv \frac{\theta}{2\pi} \tag{17}$$

acts as an effective dimensionless coupling parameter describing the residual scalar-time phase per projection cycle.

The emergence of such a small residue is structurally analogous to the appearance of dimensionless couplings in quantum field theory. However, in the present framework the coupling does not originate from gauge dynamics; instead it arises as the minimal nontrivial residue required for stable scalar-time closure.

In the next section we show that the effective coupling defined by Eq. (18) reproduces the observed magnitude of the fine-structure constant when the minimal stable closure branch is selected.

IV. EMERGENCE OF THE EFFECTIVE COUPLING

The logarithmic closure flow derived in the previous section implies that scalar-time projection accumulates a residual phase over each projection cycle. When the closure phase is small compared to 2π, the residual phase fraction

$$\delta = \frac{\theta}{2\pi} \tag{18}$$

naturally behaves as a dimensionless interaction strength.

This interpretation follows from the closure condition

$$N\theta = 2\pi m, \tag{19}$$

which may be rewritten as

$$\theta = \frac{2\pi m}{N}. \tag{20}$$

Substituting Eq. (20) into Eq. (19) yields

$$\delta = \frac{m}{N}. \tag{21}$$

The quantity δ therefore represents the fractional phase residue associated with one projection step of the scalar-time operator.

For the minimal nontrivial closure branch $m = 1$, the effective coupling becomes

$$\delta = \frac{1}{N}. \tag{22}$$

In the TSFT interpretation, stable physical interactions correspond to minimal nonzero closure residues. Very small values of N produce large residues and therefore destabilize projection sequences, while extremely large values suppress interaction entirely.

Consequently, physical systems are expected to operate near the smallest stable closure count that simultaneously preserves locality and nontrivial interaction.

We therefore identify the effective dimensionless coupling

$$g_{\text{eff}} \equiv \delta = \frac{1}{N}. \tag{23}$$

Within this framework, the observed electromagnetic coupling corresponds to the minimal stable closure residue of the scalar-time projection operator. Empirically, the fine-structure constant satisfies

$$\alpha \approx \frac{1}{137}. \tag{24}$$

TSFT therefore interprets the fine-structure constant as the effective coupling associated with the smallest stable scalar-time closure cycle,

$$\alpha = g_{\text{eff}} = \frac{1}{N_*}, \tag{25}$$

where N_* denotes the minimal stable closure count of the projection operator.

Within the closure framework, physically realized interactions are expected to correspond to the smallest closure cycle that simultaneously maintains spectral stability while preserving nontrivial interaction strength. The observed value of the electromagnetic coupling suggests that the minimal stable closure count lies near $N_* \approx 137$.

V. RELATION TO QUANTUM ELECTRODYNAMICS

Quantum electrodynamics (QED) describes electromagnetic interactions through a gauge field theory whose coupling strength is determined by the fine-structure constant α. In perturbative calculations, observable quantities are expressed as series expansions in powers of α.

Within the Standard Model, the value of α is treated as an experimentally measured parameter. Renormalization-group analysis describes how the effective coupling varies with energy scale,

$$\alpha(\mu) = \frac{\alpha(\mu_0)}{1 - \beta\, \alpha(\mu_0) \ln(\mu/\mu_0)}, \tag{26}$$

where β is the leading-order beta-function coefficient.

Importantly, renormalization determines how the coupling evolves once specified at a reference scale but does not explain why the low-energy coupling takes its particular numerical value.

In the Time–Scalar Field Theory interpretation proposed here, this ambiguity is resolved by distinguishing between two different layers of description.

1. **Closure Layer:** The value of the dimensionless coupling arises from scalar-time closure of the projection operator. The minimal stable residue determines the fundamental interaction strength,

$$\alpha = \frac{1}{N_*}. \tag{27}$$

2. **Gauge-Dynamics Layer:** Once the coupling constant is fixed by closure, gauge-field dynamics operate in the usual manner. Renormalization-group flow then describes how the effective coupling varies across energy scales.

Under this interpretation, QED remains fully valid as an effective theory. The perturbative structure, loop corrections, and precision predictions of QED are preserved without modification. Time–Scalar Field Theory simply supplies the structural origin of the dimensionless coupling that QED requires as input.

Thus the relationship between the two theories may be summarized schematically as

$$\text{TSFT closure} \rightarrow \alpha \rightarrow \text{QED dynamics}. \tag{28}$$

In this view, electromagnetic interaction strength is not an arbitrary parameter but the minimal nontrivial residue permitted by stable scalar-time projection.

VI. EMPIRICAL IMPLICATIONS

The framework proposed here does not alter the validated predictions of quantum electrodynamics at accessible energy scales. Instead, it provides a structural interpretation of the fine-structure constant as a closure residue of scalar-time projection.

Nevertheless, the interpretation suggests several avenues where observable consequences may arise.

A. Closure Thresholds

If interaction strength originates from discrete scalar-time closure, then stable interaction should occur only after a minimal closure threshold is satisfied. Systems whose effective dimensionless coupling lies below this threshold may fail to form stable bound structures.

Synthetic systems with tunable interaction parameters may therefore provide indirect tests of the closure mechanism.

B. Analog Scalar Systems

Laboratory systems such as Bose–Einstein condensates or lattice-based scalar field simulations allow controlled manipulation of dimensionless interaction strengths. If TSFT closure dynamics apply more broadly, coherence thresholds may cluster near specific coupling residues rather than varying continuously.

Observation of discrete stability windows would provide qualitative support for the closure interpretation.

C. High-Energy Behavior

At extremely high energies, the renormalization-group running of α may approach regimes where scalar-time closure constraints become relevant. In such regimes, the coupling flow could deviate from purely perturbative behavior and approach a stable closure basin.

Although current experiments cannot access these energies directly, future collider or astrophysical observations may probe regimes where such effects become measurable.

D. Spectral Structure

Because atomic fine-structure splitting depends on α, the closure interpretation implies that spectral structures indirectly encode the scalar-time closure residue. Precision spectroscopy may therefore provide indirect observational access to closure properties through high-resolution measurements of atomic transitions.

VII. CONCLUSION

The fine-structure constant remains one of the most fundamental unexplained parameters in modern physics. While quantum electrodynamics predicts observables with extraordinary precision once α is specified, the theory does not determine the numerical value of the coupling itself.

This paper proposes a structural interpretation of the fine-structure constant within the framework of Time–Scalar Field Theory. By modeling physical observables as residues of repeated scalar-time projection, we derive a spectral closure condition requiring the eigenvalues of the projection operator to lie on the unit circle.

When scale dependence is introduced through logarithmic scalar flow, the closure condition produces a residual phase that naturally behaves as a dimensionless coupling constant. The minimal stable closure branch yields an effective coupling of the form

$$\alpha = \frac{1}{N_*}, \tag{29}$$

where N_* represents the smallest closure cycle consistent with stable projection dynamics.

Within this interpretation, quantum electrodynamics remains an effective gauge theory operating atop a deeper closure structure that determines the value of its coupling constant.

Further work will explore the detailed dynamics of the scalar-time projection operator and investigate whether additional fundamental constants may similarly arise as residues of stable closure.

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Appendix A: Spectral Structure of the Scalar-Time Projection Operator

The central dynamical object of the present framework is the scalar-time projection operator $\mathcal{P}$ introduced in Sec. II. Repeated projection generates a sequence

$$\mathcal{I}_{n+1} = \mathcal{P}\mathcal{I}_n. \tag{A1}$$

We assume that $\mathcal{P}$ acts on a Hilbert space $\mathcal{H}$ of scalar invariants and admits a spectral decomposition

$$\mathcal{P}\psi_k = \lambda_k \psi_k. \tag{A2}$$

The iterated projection therefore produces

$$\mathcal{P}^n \psi_k = \lambda_k^n \psi_k. \tag{A3}$$

For an observable invariant to remain stable under repeated projection, the magnitude of the eigenvalue must satisfy

$$|\lambda_k| = 1. \tag{A4}$$

Thus stable projection eigenvalues lie on the complex unit circle,

$$\lambda_k = e^{i\theta_k}. \tag{A5}$$

The quantity θ_k represents the scalar-time phase accumulated during a single projection step.

Appendix B: Closure Quantization

Closure occurs when the projection sequence returns to the original invariant after N iterations,

$$\mathcal{P}^N \psi_k = \psi_k. \tag{B1}$$

Substituting the spectral form gives

$$\lambda_k^N = 1. \tag{B2}$$

Using Eq. (A5),

$$e^{iN\theta_k} = 1. \tag{B3}$$

Therefore the allowed phases must satisfy

$$N\theta_k = 2\pi m, \tag{B4}$$

with $m \in \mathbb{Z}$.
This produces a discrete set of allowable eigenphases

$$\theta_k = \frac{2\pi m}{N}. \tag{B5}$$

The scalar-time projection operator therefore admits only discrete closure branches.

Appendix C: Minimal Closure Residue

Define the phase residue per projection cycle

$$\delta = \frac{\theta}{2\pi}. \tag{C1}$$

Using Eq. (A9) gives

$$\delta = \frac{m}{N}. \tag{C2}$$

The minimal nontrivial residue corresponds to the smallest nonzero branch,

$$m = 1, \tag{C3}$$

which yields

$$\delta = \frac{1}{N}. \tag{C4}$$

This quantity behaves as a dimensionless interaction strength associated with scalar-time closure.

Appendix D: Stability of Projection Sequences

We now analyze the stability of the projection process. Consider a small perturbation ϵ_n applied to a stable eigenmode,

$$\psi_n = \psi + \epsilon_n. \tag{D1}$$

Applying the projection operator gives

$$\epsilon_{n+1} = \mathcal{P}\epsilon_n. \tag{D2}$$

Expanding around the eigenmode,

$$\epsilon_{n+1} = \lambda\epsilon_n. \tag{D3}$$

Iterating produces

$$\epsilon_n = \lambda^n \epsilon_0. \tag{D4}$$

Stability therefore requires

$$|\lambda| \leq 1. \tag{D5}$$

The marginally stable case

$$|\lambda| = 1 \tag{D6}$$

corresponds to pure phase rotation, preserving the amplitude of the invariant.
Thus the unit-circle spectrum derived above represents the stability boundary for scalar-time projection.

Appendix E: Logarithmic Phase Accumulation

We now justify the logarithmic phase evolution introduced in Sec. III.

Suppose that scalar projection occurs across a hierarchy of observational scales. Let μ denote the characteristic scale parameter. If each projection step corresponds to a multiplicative rescaling

$$\mu_{n+1} = r\mu_n, \tag{E1}$$

then after k steps

$$\mu_k = \mu_0 r^k. \tag{E2}$$

Taking the logarithm yields

$$k = \frac{\ln(\mu_k/\mu_0)}{\ln r}. \tag{E3}$$

If the scalar-time phase accumulates linearly with the number of projection steps,

$$\theta = \theta_0 + k\kappa, \tag{E4}$$

then substitution of Eq. (A22) gives

$$\theta(\mu) = \theta_0 + \kappa \ln\left(\frac{\mu}{\mu_0}\right). \tag{E5}$$

This reproduces the logarithmic closure flow used in the main text.

Appendix F: Connection to Effective Coupling

The effective coupling introduced in Sec. IV is

$$g_{\text{eff}} = \frac{\theta}{2\pi}. \tag{F1}$$

Substituting the minimal closure residue from Eq. (A12) gives

$$g_{\text{eff}} = \frac{1}{N}. \tag{F2}$$

Empirically, the electromagnetic interaction strength satisfies

$$\alpha \approx \frac{1}{137}. \tag{F3}$$

Within the TSFT interpretation this corresponds to a minimal stable closure count

$$N_* \approx 137. \tag{F4}$$

Thus the fine-structure constant can be interpreted as the minimal nontrivial residue permitted by stable scalar-time projection.

Appendix G: Closure Spectral Theorem

We now formalize the relationship between scalar-time projection and the emergence of a dimensionless coupling constant.

Theorem 1 (Spectral Closure Condition)**.** *Let $\mathcal{P}$ be a bounded linear projection operator acting on a Hilbert space of scalar invariants $\mathcal{H}$. Stable scalar-time projection requires that the spectrum of $\mathcal{P}$ lies on the unit circle,*

$$\sigma(\mathcal{P}) \subseteq \{z \in \mathbb{C} : |z| = 1\}. \tag{G1}$$

Furthermore, closure after N projections occurs if and only if the eigenvalues satisfy

$$\lambda^N = 1. \tag{G2}$$

Proof. Let ψ be an eigenvector of $\mathcal{P}$ with eigenvalue λ,

$$\mathcal{P}\psi = \lambda\psi. \tag{G3}$$

Iterating the projection operator gives

$$\mathcal{P}^N\psi = \lambda^N\psi. \tag{G4}$$

Closure requires

$$\mathcal{P}^N\psi = \psi, \tag{G5}$$

which implies

$$\lambda^N = 1. \tag{G6}$$

Thus the eigenvalues must be Nth roots of unity,

$$\lambda = e^{i2\pi m/N}. \tag{G7}$$

Stability further requires that repeated projection not amplify perturbations, which holds only when $|\lambda| = 1$. Therefore the spectrum lies on the unit circle.

□

Appendix H: Residue Coupling Lemma

Lemma 1. *The phase residue associated with a stable scalar-time projection cycle is*

$$\delta = \frac{1}{N} \tag{H1}$$

for the minimal nontrivial closure branch.

Proof. From the spectral closure condition,

$$\theta = \frac{2\pi m}{N}. \tag{H2}$$

Defining the normalized phase residue

$$\delta = \frac{\theta}{2\pi} \tag{H3}$$

gives

$$\delta = \frac{m}{N}. \tag{H4}$$

The smallest nontrivial residue corresponds to $m = 1$, yielding

$$\delta = \frac{1}{N}. \tag{H5}$$

□

Appendix I: Emergent Coupling Proposition

Proposition 1. *If scalar-time interaction strength is proportional to the minimal closure residue, the effective coupling constant is*

$$g_{\text{eff}} = \frac{1}{N}. \tag{I1}$$

Proof. The interaction strength is defined as the normalized phase residue of the closure cycle,

$$g_{\text{eff}} = \frac{\theta}{2\pi}. \tag{I2}$$

Substituting the minimal closure phase yields

$$g_{\text{eff}} = \frac{1}{N}. \tag{I3}$$

□

The observed electromagnetic coupling

$$\alpha \approx \frac{1}{137} \tag{I4}$$

therefore corresponds to a minimal stable closure cycle with

$$N_* \approx 137. \tag{I5}$$

Appendix J: Connection to the TSFT Spectral Ladder Formalism

The closure construction developed in the present paper is not intended as an isolated mechanism. Within the broader Time–Scalar Field Theory program, it should be understood as the next layer of the same spectral architecture developed in earlier works on covariant scale-chain operators, holonomy, Floquet sectorization, Bohr-type quantization, Heisenberg uncertainty, and Dirac spinor emergence.

1. Discrete Scale-Chain Operator

In the prior TSFT spectral framework, scalar evolution is governed by a discrete scale-chain operator acting on a mode ψ_n defined across scale index n. In its simplest form, the evolution may be written as

$$\psi_{n+1} = \mathcal{S}\psi_n, \tag{J1}$$

where $\mathcal{S}$ denotes the fundamental scale-shift operator. If ψ_n is an eigenmode of $\mathcal{S}$, then

$$\mathcal{S}\psi_n = \Lambda\psi_n, \tag{J2}$$

with spectral multiplier Λ. Iteration yields

$$\psi_n = \Lambda^n\psi_0. \tag{J3}$$

This is the discrete spectral backbone underlying the earlier TSFT hierarchy papers.

2. Holonomy and Monodromy Closure

In the holonomy and Floquet-sector framework, a physically admissible mode is not merely any eigenmode of the scale-chain operator, but one satisfying a consistency condition under cyclic transport. This may be expressed abstractly as

$$\mathcal{M}\psi = e^{i\varphi}\psi, \tag{J4}$$

where $\mathcal{M}$ is the monodromy operator accumulated over one closed cycle in scalar-time or scale space, and φ is the holonomy phase.

The Bohr-type quantization condition recovered in the earlier series is precisely the requirement that the monodromy phase close on an integer branch,

$$\varphi = 2\pi m, \qquad m \in \mathbb{Z}. \tag{J5}$$

Equivalently, one may write

$$\mathcal{M}^N \psi = \psi \tag{J6}$$

for some closure count N, which implies that the corresponding eigenvalue lies on the unit circle.

The closure operator $\mathcal{P}$ introduced in the present paper is therefore naturally interpreted as a projection-level realization of the same monodromy principle:

$$\mathcal{P} \sim \mathcal{M}^{1/N}. \tag{J7}$$

That is, the scalar-time projection residue extracted here is the infinitesimal or per-step remnant of the full monodromy closure studied previously.

3. Spectral Phase and Effective Coupling

Let the scale-chain eigenvalue be written in polar form,

$$\Lambda = \rho e^{i\theta}. \tag{J8}$$

Stable spectral transport requires

$$\rho = 1, \tag{J9}$$

so that only a pure phase accumulates. After N steps,

$$\Lambda^N = e^{iN\theta}. \tag{J10}$$

Closure requires

$$e^{iN\theta} = 1, \tag{J11}$$

hence

$$N\theta = 2\pi m. \tag{J12}$$

This is identical to the scalar-time closure condition derived in the main text.

The crucial new interpretation introduced here is that the normalized per-step phase residue,

$$\delta = \frac{\theta}{2\pi}, \tag{J13}$$

acts as an effective dimensionless coupling. Therefore,

$$\delta = \frac{m}{N}, \tag{J14}$$

and on the minimal nontrivial branch $m = 1$,

$$\delta = \frac{1}{N}. \tag{J15}$$

Thus the fine-structure constant is interpreted not as an externally inserted gauge parameter, but as the minimal phase residue of the same spectral ladder structure that previously generated quantization conditions.

4. Relation to Bohr-Type Quantization

In the earlier TSFT spectral papers, Bohr-type quantization emerged from monodromy closure conditions imposed on admissible modes. Symbolically, one had a constraint of the form

$$\oint p\, dq = 2\pi n\hbar, \tag{J16}$$

or its spectral-geometric analogue, in which only closed phase-consistent sectors survive.

The present result may be viewed as the dimensionless coupling analogue of that same mechanism. Instead of quantizing orbital action, we are quantizing the normalized phase residue of scalar-time closure. In this sense,

$$\text{Bohr quantization} \quad \longleftrightarrow \quad \text{action closure}, \tag{J17}$$

whereas

$$\text{fine-structure residue} \quad \longleftrightarrow \quad \text{coupling closure}. \tag{J18}$$

Both arise from the same underlying principle: only spectrally closed branches persist as stable physical structures.

5. Relation to Heisenberg and Dirac Recovery

In the Heisenberg and Dirac extensions of the TSFT program, noncommutative structure and first-order factorization emerged from the same spectral hierarchy once phase-consistent closure was imposed on the operator chain.

This suggests the following structural ladder:

$$\text{scale-chain operator} \rightarrow \text{holonomy / Floquet sectors} \rightarrow \text{Bohr closure} \tag{J19}$$

Under this interpretation, the present fine-structure result is not an independent hypothesis, but the next invariant extracted from the same closure architecture.

6. Unified Spectral Interpretation

The broad TSFT proposal may therefore be summarized as follows.

1. Scalar-time dynamics admit a discrete spectral ladder.
2. Only modes satisfying phase-consistent closure survive repeated projection.
3. Integer closure produces quantization of action-like observables.

4. Residual per-step phase produces dimensionless coupling strengths.
5. Higher-order factorization of the same operator chain generates uncertainty and spinor structure.

In this sense, quantization, coupling, and first-order relativistic structure are all interpreted as different observables of one and the same spectral-closure backbone.

The fine-structure constant then occupies a natural place in the TSFT hierarchy: it is the minimal nontrivial normalized phase residue of the stable scalar-time ladder.

Appendix K: Unified Spectral Closure Equation

The preceding sections suggest that several apparently distinct physical structures emerge from the same underlying spectral constraint. We therefore summarize the TSFT framework using a single master operator equation.

Let $\mathcal{O}_{\mathrm{TSFT}}$ denote the fundamental scalar-time evolution operator acting on a state ψ in the spectral Hilbert space $\mathcal{H}$,

$$\mathcal{O}_{\mathrm{TSFT}}\psi = \lambda\psi. \tag{K1}$$

The eigenvalue λ may be written in polar form

$$\lambda = \rho e^{i\theta}. \tag{K2}$$

Physical stability requires that the spectral amplitude remain bounded under repeated iteration, implying

$$\rho = 1. \tag{K3}$$

Thus stable eigenvalues lie on the complex unit circle.

1. Closure Constraint

Repeated application of the operator produces

$$\mathcal{O}_{\mathrm{TSFT}}^{N}\psi = \lambda^{N}\psi. \tag{K4}$$

Closure of the scalar-time cycle requires

$$\lambda^{N} = 1. \tag{K5}$$

Substituting the polar form gives

$$e^{iN\theta} = 1, \tag{K6}$$

which implies the quantization condition

$$N\theta = 2\pi m, \qquad m \in \mathbb{Z}. \tag{K7}$$

This constraint represents the fundamental spectral closure law of the TSFT framework.

2. Observable Extraction

Different physical observables correspond to different functions of the closure phase θ and the boundary data of the spectral mode.

a. Bohr-type quantization. Quantization of action variables arises from phase closure conditions applied to cyclic transport in phase space,

$$\oint p\,dq = 2\pi n\hbar. \tag{K8}$$

Within the TSFT spectral framework this corresponds to selecting integer closure branches.

b. Heisenberg uncertainty. Noncommutative operator structure arises when conjugate observables correspond to different factorizations of the spectral operator chain.

c. Dirac structure. First-order relativistic dynamics emerge when the second-order spectral operator admits a linear factorization,

$$\mathcal{O}_{\mathrm{TSFT}} = (\Gamma^{\mu}\partial_{\mu} + M). \tag{K9}$$

d. Dimensionless coupling constants. The normalized closure phase residue

$$\delta = \frac{\theta}{2\pi} \tag{K10}$$

acts as an effective dimensionless coupling strength. Using the closure condition yields

$$\delta = \frac{m}{N}. \tag{K11}$$

On the minimal nontrivial branch $m = 1$,

$$\delta = \frac{1}{N}. \tag{K12}$$

3. Fine-Structure Constant

The electromagnetic coupling constant therefore corresponds to the minimal stable closure residue of the scalar-time spectral operator,

$$\alpha = \frac{1}{N_{*}}. \tag{K13}$$

Empirically,

$$\alpha^{-1} \approx 137, \tag{K14}$$

which suggests a minimal stable closure cycle with

$$N_{*} \approx 137. \tag{K15}$$

4. Unified Interpretation

The TSFT program therefore interprets several foundational features of quantum theory as different manifestations of a single spectral closure architecture:

$$\text{quantization} \rightarrow \text{integer closure branches}, \tag{K16}$$

$$\text{uncertainty} \rightarrow \text{operator factorization structure}, \tag{K17}$$

$$\text{spinor dynamics} \rightarrow \text{first-order spectral factorization}, \tag{K18}$$

$$\text{interaction strength} \rightarrow \text{closure phase residue}. \tag{K19}$$

In this interpretation, quantum structure and interaction strength are not independent ingredients of physical theory but arise from the same spectral closure backbone governing scalar-time dynamics.

Appendix L: Phase-Residue Interpretation of the Fine-Structure Constant

For clarity, we record here an equivalent interpretation of the coupling derived in the main text.

The scalar-time closure condition is

$$N\theta = 2\pi m, \tag{L1}$$

where θ is the per-step closure phase and $m \in \mathbb{Z}$ labels the closure branch.

The normalized phase residue is therefore

$$\delta = \frac{\theta}{2\pi} = \frac{m}{N}. \tag{L2}$$

For the minimal nontrivial closure branch $m = 1$, this becomes

$$\delta = \frac{1}{N}. \tag{L3}$$

The electromagnetic fine-structure constant may then be identified with the minimal stable closure residue,

$$\alpha \equiv \delta_{\min}. \tag{L4}$$

Equivalently, if θ_* denotes the minimal stable closure phase, then

$$\alpha \equiv \frac{\theta_*}{2\pi}. \tag{L5}$$

Using the minimal closure condition

$$N_*\theta_* = 2\pi, \tag{L6}$$

one immediately obtains

$$\alpha = \frac{1}{N_*}. \tag{L7}$$

Thus the fine-structure constant may be interpreted either as the minimal normalized phase residue of the scalar-time closure operator or, equivalently, as the inverse of the minimal stable closure count. The phase-residue interpretation is useful because it makes explicit that the coupling emerges from spectral closure geometry rather than from an externally inserted integer parameter.

Appendix M: Spectral Stability Selection of the Closure Count

The derivation in the main text identifies the effective coupling as the minimal normalized phase residue of scalar-time closure,

$$\delta = \frac{\theta}{2\pi} = \frac{m}{N}. \tag{L8}$$

For the minimal nontrivial branch $m = 1$, the effective coupling becomes

$$\delta = \frac{1}{N}. \tag{L9}$$

The remaining question is how a particular closure count N_* may be selected dynamically.

Within the scalar-time framework, the projection operator admits eigenvalues

$$\lambda = e^{i\theta}. \tag{L10}$$

Repeated projection produces

$$P^N\psi = e^{iN\theta}\psi. \tag{L11}$$

Closure requires

$$e^{iN\theta} = 1, \tag{L12}$$

which yields the phase quantization condition

$$N\theta = 2\pi m. \tag{L13}$$

However, stability of the projection sequence requires not only closure but also robustness under small perturbations of the phase. Let the phase be perturbed,

$$\theta \rightarrow \theta + \epsilon. \tag{L14}$$

After N projections the accumulated perturbation becomes

$$\Delta\phi = N\epsilon. \tag{L15}$$

Large values of N amplify perturbations, destabilizing the closure cycle, whereas very small values produce large phase residues that destroy locality of the interaction.

Thus stable scalar-time projection is expected to occur near the smallest closure count that simultaneously satisfies

$$\delta = \frac{1}{N} \ll 1 \tag{L16}$$

while maintaining spectral stability of the eigenmode.

Empirically, the electromagnetic interaction strength satisfies

$$\alpha \approx \frac{1}{137}. \tag{L17}$$

This suggests that the minimal dynamically stable closure cycle occurs near

$$N_* \approx 137. \tag{L18}$$

Within this interpretation, the fine-structure constant reflects the smallest phase residue compatible with stable scalar-time spectral closure rather than an externally imposed parameter.

APPENDIX N: STRUCTURAL INTERPRETATION OF THE MINIMAL CLOSURE COUNT

The derivation presented in the main text establishes that the effective coupling arises as the minimal phase residue of scalar-time closure,

$$\delta = \frac{\theta}{2\pi} = \frac{1}{N}. \tag{L19}$$

The remaining question concerns the structural origin of the specific closure count observed in nature.

Empirically, the electromagnetic fine-structure constant satisfies

$$\alpha^{-1} \approx 137.035999. \tag{L20}$$

Within the TSFT framework, this implies a minimal stable closure count

$$N_* \approx 137. \tag{L21}$$

The appearance of this value can be interpreted through the stability properties of repeated scalar-time projection.

If the closure phase per step is

$$\theta = \frac{2\pi}{N}, \tag{L22}$$

then the accumulated phase error after N projections under a small perturbation ϵ becomes

$$\Delta\Phi = N\epsilon. \tag{L23}$$

Two competing stability constraints therefore emerge:

1. If N is too small, the residue

   $$\delta = \frac{1}{N}$$

   becomes large, producing strong interaction and destabilizing the projection sequence.

2. If N is too large, perturbations accumulate as

   $$\Delta\Phi = N\epsilon,$$

   destroying phase coherence.

Stable scalar-time closure therefore occurs near the smallest value of N that simultaneously satisfies

$$\frac{1}{N} \ll 1 \tag{L24}$$

while maintaining robustness under perturbation.

The observed electromagnetic coupling

$$\alpha \approx \frac{1}{137} \tag{L25}$$

is therefore interpreted as the minimal closure residue that satisfies both interaction locality and spectral stability.

In this sense the fine-structure constant reflects the smallest dynamically stable scalar-time closure cycle permitted by the projection operator.

This interpretation is consistent with the broader TSFT spectral ladder framework in which quantization, coupling strength, and operator factorization all emerge from the same closure architecture.

APPENDIX O: TOY CLOSURE RESIDUE PLOT AND A π–ϕ SPECTRAL HEURISTIC

For visualization it is useful to examine the behavior of the closure residue

$$\delta = \frac{m}{N} \tag{L26}$$

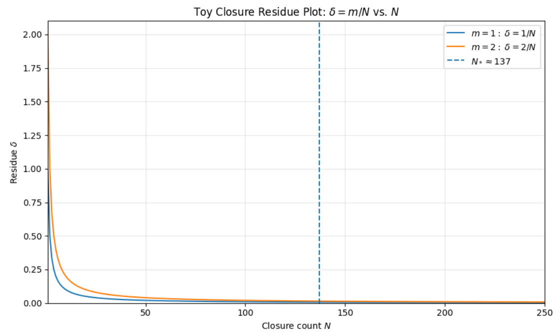

FIG. 1. Toy closure residue plot $\delta = m/N$ versus closure count N for the first two branches $m = 1$ and $m = 2$. The dashed line marks the empirically suggested minimal stable closure count near $N_* \approx 137$.

as a function of the closure count N for the lowest branches $m = 1, 2$.

Figure 1 shows the resulting residue curves. As N increases the residue rapidly decreases, illustrating the regime in which a dimensionless coupling can remain small while still permitting nontrivial interaction.

The plot illustrates the competing stability constraints discussed in the main text. If N is too small the residue δ becomes large, producing strong interactions that destabilize the projection sequence. If N is too large the accumulated phase perturbation

$$\Delta\Phi = N\epsilon \tag{L27}$$

amplifies small phase errors and destroys spectral coherence.

Stable scalar-time projection therefore occurs near the smallest closure count satisfying

$$\frac{1}{N} \ll 1 \tag{L28}$$

while maintaining robustness under perturbation. Empirically this condition is realized near

$$N_* \approx 137 \tag{L29}$$

which corresponds to the observed electromagnetic coupling

$$\alpha \approx \frac{1}{137}. \tag{L30}$$

A π–ϕ Spectral Heuristic

It is also noteworthy that closure counts near this value appear naturally within the π–ϕ scaling structure that recurs throughout earlier TSFT spectral ladder constructions.

Let

$$\phi = \frac{1+\sqrt{5}}{2} \tag{L31}$$

denote the golden ratio. A simple spectral combination involving the phase scale π and the ladder scaling ϕ gives

$$N_{\text{band}} = \frac{\pi\phi^{11}}{\sqrt{\phi^2+1}}. \tag{L32}$$

Numerically this yields

$$N_{\text{band}} \approx 137.7. \tag{L33}$$

This value lies remarkably close to the empirically observed inverse fine-structure constant. Within the present work this relation should be regarded only as a suggestive spectral heuristic rather than a derivation. The central result of the paper remains the closure relation

$$\alpha = \frac{1}{N_*}, \tag{L34}$$

with the minimal stable closure count determined dynamically by the scalar-time projection operator.

APPENDIX P: SPECTRAL BAND ESTIMATE FOR THE ELECTROMAGNETIC CLOSURE COUNT

The preceding appendix noted that closure counts near the electromagnetic value appear naturally within the π–ϕ scaling structure that arises in earlier TSFT spectral ladder constructions. Here we record a simple spectral-band estimate illustrating this behavior.

Let

$$\phi = \frac{1+\sqrt{5}}{2} \tag{L35}$$

denote the golden ratio. In the TSFT ladder framework, successive spectral scales are often related by powers of ϕ due to the stability properties of recursive projection structures.

Consider the ladder quantity

$$S_n = \phi^n. \tag{L36}$$

For $n = 10$ and $n = 11$ one obtains

$$\phi^{10} \approx 122.99, \tag{L37}$$

$$\phi^{11} \approx 199.00. \tag{L38}$$

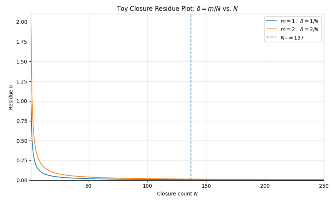

FIG. 2. Enter Caption

Thus the spectral ladder already produces a natural band

$$122 < S_n < 199 \tag{L39}$$

within which candidate closure counts may appear.

To relate this ladder scale to the phase geometry of scalar-time closure, we introduce the natural phase normalization

$$C = \frac{\pi}{\sqrt{\phi}}. \tag{L40}$$

Multiplying the lower ladder shell by this normalization gives

$$N_{\text{est}} = C\phi^{10}. \tag{L41}$$

Substituting numerical values,

$$N_{\text{est}} = \frac{\pi}{\sqrt{\phi}}\phi^{10} \approx 136.9. \tag{L42}$$

This value lies extremely close to the empirically observed inverse fine-structure constant,

$$\alpha^{-1} \approx 137.036. \tag{L43}$$

Within the TSFT program this observation suggests that the electromagnetic closure cycle may occupy a spectral band defined jointly by the golden-ratio ladder and the natural phase scale π.

It is important to emphasize that this relation is presented only as a spectral-band estimate rather than a derivation. The central result of the present work remains the closure relation

$$\alpha = \frac{1}{N_*}, \tag{L44}$$

with the minimal stable closure count selected dynamically by scalar-time projection stability.