# Rivet Selection and Mass Dictionary in Time–Scalar Field Theory

## A Bridge from TSFT Spectral Geometry to Particle Mass Predictions

Jordan G. Farrell
The Zebra Journal of Unified Physics (ZJUP)
ORCID: 0009-0002-2171-809X
`(add preferred contact/email here)`

February 2026

**Abstract**

We present the missing operational bridge between the Time–Scalar Field Theory (TSFT) spectral-geometry program and explicit particle-mass prediction. Prior papers in the TSFT spectral series establish: (i) a self-adjoint scale-chain generator and its $\mathrm{SU}(N)$-covariant extensions; (ii) Floquet sectorization and holonomy reduction; (iii) Bohr-type discretization via monodromy closure and half-line admissibility; (iv) an exact Weyl-pair structure yielding a Heisenberg-type uncertainty relation; (v) first-order Dirac-type factorization and a controlled Schrödinger limit on low spectral windows; and (vi) the Born probability rule from additive, noncontextual projector measures. The remaining task for a predictive microphysics is a well-defined dictionary identifying which TSFT spectral objects correspond to persistent, localized particle states and how their eigenvalues map to observed rest masses.

Using the Temporal Coherence Principle (TCP) and the *rivet* concept as microphysical phase anchors, we define TCP-stable rivet modes as admissible localized eigenmodes that remain phase-locked under TSFT evolution. We then propose a calibrated mass map of the form $m_{n,\alpha} = K\sqrt{\lambda_n}\,G(\alpha)$, where $\lambda_n$ is the monodromy-selected TSFT eigenvalue, $\alpha$ is a holonomy-sector parameter entering the first-order closure, $G(\alpha)$ encodes the holonomy-induced gap structure, and $K$ is a single global scale fixed by one experimental calibration. This yields a transparent parameter count and an explicit protocol for producing dimensionless mass-ratio predictions from TSFT spectral data.

**Keywords:** Time–Scalar Field Theory; spectral geometry; rivets; temporal coherence; monodromy; holonomy; mass hierarchy; particle prediction.

## 1 Introduction

The Time–Scalar Field Theory (TSFT) program has recently developed a coherent operator-theoretic reconstruction of the core kinematic and dynamical structures of single-particle quantum mechanics. Prior work established: (i) a self-adjoint scale-chain generator and its $\mathrm{SU}(N)$-covariant extensions; (ii) Floquet sectorization and holonomy reduction; (iii) Bohr-type discretization via monodromy closure and half-line admissibility; (iv) an exact Weyl-pair structure yielding a Heisenberg-type uncertainty relation; (v) a first-order Dirac-type factorization together with a controlled Schrödinger limit on low spectral windows; and (vi) the Born probability rule derived from additive, noncontextual projector measures.

Taken together, these results demonstrate that TSFT reproduces the standard single-particle quantum kinematic framework within a unified spectral geometry. However, an explicit bridge from TSFT spectral data to observable particle masses has not yet been fully centralized in a single operational prescription. While previous papers introduced the hierarchical eigenvalue structure and identified holonomy-dependent gap effects, the precise dictionary linking admissible TSFT modes to persistent microphysical particles remains to be stated in a compact, calibrated form.

The purpose of the present work is to provide this missing bridge. Building on the Temporal Coherence Principle (TCP) and the concept of *rivets* as microphysical phase anchors, we formalize the class of TCP-stable localized eigenmodes that behave as particle-like excitations. We then introduce an explicit mass map that converts the monodromy-selected TSFT spectrum and holonomy sector data into observable rest-mass predictions with a transparent parameter count.

The main contributions of this paper are:

1. A precise definition of TCP-stable rivet modes as the TSFT microphysical carriers of particle persistence.
2. An explicit mass dictionary $m_{n,\alpha} = K\sqrt{\lambda_n}\, G(\alpha)$ linking TSFT eigenvalues and holonomy sectors to observable masses.
3. A one-parameter calibration scheme demonstrating that, after a single global scale is fixed, the remaining mass ratios are determined by TSFT spectral data.
4. A clear positioning of the result within the broader TSFT program and its current scope.

This paper is intentionally conservative in its claims. We do not yet attempt a full Standard Model embedding or interacting quantum field theory completion. Rather, the goal is to establish a clean, parameter-controlled bridge from the existing TSFT spectral machinery to the first generation of concrete mass-hierarchy predictions.

## 2 Rivet Modes and TCP Stability

We now formalize the notion of a *rivet* within the TSFT framework. Informally, rivets represent localized phase-locked excitations that remain coherent under Temporal Coherence Principle (TCP) evolution. They provide the operational bridge between the TSFT spectral construction and persistent microphysical particles.

### 2.1 TSFT spectral setting

Let $\mathcal{H}$ denote the TSFT Hilbert space and let $\mathcal{L}$ be the self-adjoint scale-chain generator obtained in the spectral-geometry construction. As established in the prior TSFT series, admissible physical modes arise from monodromy compatibility together with the appropriate boundary or half-line conditions.

We write the eigenvalue problem as

$$\mathcal{L}\psi = \lambda\psi,$$

where $\lambda$ belongs to the admissible TSFT spectrum.

### 2.2 Definition of TCP-stable rivets

We now isolate the subclass of eigenmodes that correspond to physically persistent excitations.

**Definition 1** (TCP-stable rivet mode)**.** Let $\psi \in \mathcal{H}$ be an admissible eigenmode of $\mathcal{L}$ with eigenvalue $\lambda$. We call $\psi$ a *TCP-stable rivet mode* if the following conditions hold:

1. **Spectral admissibility.** The pair $(\psi, \lambda)$ satisfies the monodromy and boundary admissibility conditions established in the TSFT spectral series.
2. **TCP persistence.** Under TSFT time evolution, the state remains norm-bounded; i.e., there exists $C > 0$ such that
$$\|\psi(t)\| \leq C \quad \text{for all admissible scalar times } t.$$
3. **Spatial localization.** The probability density $|\psi|^2$ is localized with finite second moment.

A physical particle is identified with a TCP-stable rivet mode.

### 2.3 Physical interpretation

The rivet concept encodes the idea that particle-like objects are not fundamental point entities but rather dynamically stabilized, phase-locked excitations of the underlying TSFT structure. The TCP condition ensures long-time persistence, while the spectral admissibility condition enforces the discrete structure previously derived from monodromy closure and half-line selection.

In this framework, the remaining task is to convert the admissible spectral data $(\lambda_n, \alpha)$ associated with TCP-stable rivets into observable rest masses. This mapping is constructed in the next section.

## 3 Mass Observable from TSFT Spectral Data

We now construct the operational dictionary that maps TCP-stable rivet modes to observable particle rest masses. The goal is to express the mass spectrum directly in terms of quantities already fixed by the TSFT spectral program.

### 3.1 Spectral inputs from TSFT

The TSFT construction provides the following ingredients:

- A discrete admissible spectrum $\{\lambda_n\}$ arising from monodromy closure and half-line admissibility.
- A holonomy parameter $\alpha$ entering the SU($N$)-covariant first-order (Dirac-type) factorization.
- An effective gap structure associated with the positive-energy branch of the reduced dynamics.

These objects are not introduced ad hoc; they arise naturally from the TSFT operator geometry developed in the prior series.

### 3.2 Rivet mass map

Motivated by the interpretation of rivets as localized trapped-momentum excitations of the TSFT field, we identify the rest mass of a TCP-stable mode via the mapping

$$\boxed{m_{n,\alpha} = K\,\sqrt{\lambda_n}\,G(\alpha)}$$

where:

- $K$ is a single global calibration constant,
- $\lambda_n$ is the admissible TSFT eigenvalue,
- $G(\alpha)$ encodes holonomy-induced gap structure.

The square-root dependence reflects the first-order (Dirac-type) closure of the TSFT dynamics, under which the quadratic spectral operator governs the underlying stiffness while the physical excitation scale is linear in the effective momentum.

### 3.3 Holonomy gap factor

From the TSFT first-order reduction, the effective gap takes the schematic form

$$M_{\text{eff}}(\alpha) = \sqrt{m^2 + c^2 \sin^2 \alpha}.$$

Normalizing so that the trivial holonomy sector leaves the mass unchanged, we define

$$G(\alpha) = \sqrt{1 + \beta \sin^2 \alpha},$$

where the constant $\beta$ is determined by the TSFT Dirac-sector coefficients. By construction,

$$G(0) = 1.$$

### 3.4 Interpretation

Equation above provides the missing operational bridge between the TSFT spectral hierarchy and observable particle masses. Once the admissible spectrum $\lambda_n$ and the allowed holonomy sectors $\alpha$ are specified, the only continuous freedom remaining is the single global scale $K$.

## 4 Parameter Counting and Predictive Structure

We now make explicit the number of adjustable parameters entering the TSFT mass relation and distinguish them from quantities fixed by the underlying spectral construction.

### 4.1 Continuous parameters

The mass map

$$m_{n,\alpha} = K\,\sqrt{\lambda_n}\,G(\alpha)$$

contains a single continuous calibration parameter:

- $K$ — a global scale converting TSFT units to physical mass units.

This constant is fixed once by comparison with a single experimental mass and is not subsequently retuned.

### 4.2 Structural quantities (not free)

The following quantities are determined by the TSFT framework and are therefore not adjustable parameters:

- the geometric ratio $\phi$ entering the scale-chain construction,
- the admissibility and monodromy constraints selecting $\lambda_n$,
- the Weyl-pair structure underlying the uncertainty relation,
- the Clifford/Dirac closure relations,
- the Born-rule normalization of probabilities.

### 4.3 Discrete labels

In addition to the single continuous scale, the spectrum depends on discrete identifiers:

- the mode index $n$ labeling admissible eigenvalues,
- the holonomy sector $\alpha$.

These are not tunable continuous parameters but structural choices arising from the TSFT spectral geometry.

**Predictive statement.** After a single calibration of the global scale $K$, all remaining mass ratios are determined by TSFT spectral data.

## 5 Calibration and Dimensionless Mass Ratios

We now implement the single-parameter calibration and derive the resulting dimensionless mass relations implied by the TSFT spectral construction.

### 5.1 Single-scale calibration

The global constant $K$ is fixed by identifying one TCP-stable rivet mode with a known reference mass. For definiteness, we take the lightest stable mode to correspond to the electron mass $m_e$:

$$K = \frac{m_e}{\sqrt{\lambda_{n_0}}\, G(\alpha_0)},$$

where $(n_0, \alpha_0)$ labels the reference rivet mode.

### 5.2 Predicted hierarchy

Substituting the calibrated value of $K$ into the general mass map yields

$$m_{n,\alpha} = m_e \, \frac{\sqrt{\lambda_n}\, G(\alpha)}{\sqrt{\lambda_{n_0}}\, G(\alpha_0)}.$$

Thus the TSFT framework predicts the dimensionless mass ratios

$$\boxed{\frac{m_{n,\alpha}}{m_e} = \frac{\sqrt{\lambda_n}\, G(\alpha)}{\sqrt{\lambda_{n_0}}\, G(\alpha_0)}}.$$

### 5.3 Remarks on predictive content

The structure above shows that, once the admissible spectrum $\lambda_n$ and the allowed holonomy sectors are specified, the hierarchy of masses follows without further continuous tuning. The TSFT program therefore reduces the mass spectrum problem to the determination of:

1. the admissible eigenvalue ladder $\{\lambda_n\}$,
2. the physically realized holonomy sectors,
3. the identification of the reference mode.

Explicit numerical evaluation of the lowest TSFT modes and their associated holonomy splittings will be reported in subsequent work.

## 6 Falsifiable Signatures and Near-Term Tests

A central objective of the TSFT program is the production of experimentally distinguishable consequences. While the present work focuses on the structural mass dictionary, the framework already implies several near-term falsifiable signatures.

### 6.1 Spectral spacing structure

Because admissible eigenvalues arise from monodromy closure together with the geometric attenuation of the scale chain, the low-lying spectrum is expected to obey a constrained spacing law of the schematic form

$$\lambda_n \sim n^2 \, \phi^{-N} \quad \text{(low-spectrum regime)},$$

up to calculable corrections determined by the detailed TSFT boundary conditions. Precise numerical evaluation of the first few admissible modes provides a direct internal consistency check of the framework.

### 6.2 Holonomy splitting pattern

The mass relation

$$m_{n,\alpha} = K \sqrt{\lambda_n} \, G(\alpha)$$

predicts that modes sharing the same spectral index $n$ but differing in holonomy sector $\alpha$ will exhibit controlled mass splittings governed by the function $G(\alpha)$. Observation of a regular holonomy-like splitting pattern within a particle family would provide support for the TSFT mechanism.

### 6.3 Quartic dispersion fingerprint

Prior TSFT analysis yields the exact operator identity relating the scale-momentum operator $P_s$ to the underlying chain generator. In the low-momentum regime, this produces a characteristic quartic correction to the effective dispersion relation. Detection of such a locked quartic term—beyond generic lattice artifacts—would constitute a distinctive TSFT signature.

### 6.4 Parameter economy

Perhaps the most immediate falsifiable feature is the parameter economy of the construction. After a single global calibration, the remaining mass ratios are fixed by TSFT spectral data. Failure of the computed hierarchy to align with observed mass patterns, once the admissible spectrum and holonomy sectors are specified, would rule out the present form of the model.

### 6.5 Outlook

Detailed numerical evaluation of the admissible TSFT spectrum and its holonomy structure is currently in progress and will be reported in forthcoming work. Those calculations will provide the first quantitative tests of the mass-hierarchy mechanism proposed here.

## 7 Scope and Limitations

The purpose of the present work is narrowly defined: to establish a transparent operational bridge between the TSFT spectral construction and observable particle mass hierarchies. While the results provide a structured and parameter-economical framework for mass generation, several important extensions remain outside the current scope.

### 7.1 No full Standard Model embedding

We do not claim in this paper to derive the full Standard Model particle content or gauge structure. In particular, the identification of specific holonomy sectors with known fermion families, as well as the incorporation of gauge interactions, requires additional dynamical development.

### 7.2 No interacting quantum field theory completion

The present analysis operates at the level of single-particle spectral structure. A full interacting quantum field theory formulation within TSFT, including scattering amplitudes, renormalization behavior, and multi-particle sectors, remains to be constructed.

### 7.3 Holonomy sector selection

While the mass map

$$m_{n,\alpha} = K\sqrt{\lambda_n}\,G(\alpha)$$

makes explicit how holonomy enters the mass hierarchy, the dynamical mechanism selecting the physically realized set of holonomy sectors has not yet been uniquely fixed. This question is under active investigation.

### 7.4 Numerical spectrum evaluation

The present work focuses on the structural mapping and parameter counting. High-precision numerical evaluation of the admissible TSFT spectrum, including boundary-condition sensitivity and holonomy splitting, will be reported in subsequent work and will provide the primary quantitative tests of the framework.

### 7.5 Interpretive caution

Accordingly, the results should be viewed as establishing a well-posed and falsifiable pathway from TSFT spectral geometry to particle mass hierarchies, rather than as a completed microphysical theory. The framework is designed to become progressively more constrained as the admissible spectrum and holonomy structure are computed explicitly.

**Summary.** Within its stated scope, the TSFT rivet framework provides a parameter-economical mechanism for generating discrete mass hierarchies from first-principles spectral data. Its ultimate viability will be determined by forthcoming numerical and phenomenological tests.

## 8 Conclusion

We have presented a compact operational bridge between the Time–Scalar Field Theory (TSFT) spectral program and the emergence of particle mass hierarchies. Building on the previous TSFT reconstruction of quantum kinematics and dynamics, the present work formalizes TCP-stable rivet modes as the persistent microphysical excitations of the theory and introduces an explicit mass dictionary linking admissible spectral data to observable rest masses.

The central result is the calibrated relation

$$m_{n,\alpha} = K\sqrt{\lambda_n}\,G(\alpha),$$

which expresses particle masses in terms of (i) the monodromy-selected TSFT eigenvalues and (ii) the holonomy sector entering the first-order closure. The construction makes the parameter structure transparent: after a single global calibration, the remaining mass ratios are fixed by TSFT spectral data.

Equally important, the framework yields concrete avenues for falsification. The admissible eigenvalue ladder, the holonomy-induced splitting pattern, and the characteristic low-momentum dispersion structure together provide a set of quantitative targets for numerical and phenomenological testing. The viability of the TSFT rivet mechanism therefore rests on explicit spectral computations rather than on adjustable multi-parameter fits.

Within its stated scope, this work establishes that the TSFT spectral geometry is sufficiently developed to support a well-posed and parameter-economical pathway to particle mass generation. The next stage of the program will focus on high-precision evaluation of the admissible TSFT spectrum and the resulting dimensionless mass hierarchy, enabling direct comparison with observed particle data.

**Outlook.** The TSFT framework now stands at the threshold of quantitative phenomenology. Detailed numerical studies of the admissible spectrum and holonomy structure will determine whether the rivet-based mass mechanism can successfully reproduce the observed hierarchy of elementary particle masses.

## References

[1] J. G. Farrell, “The Scale-Chain Operator in Time–Scalar Field Theory,” *Zebra Journal of Unified Physics (ZJUP)*, 2025.

[2] J. G. Farrell, “Holonomy and Floquet Sectorization in SU($N$)-Covariant Time–Scalar Spectral Geometry,” *ZJUP*, 2025.

[3] J. G. Farrell, "Bohr Quantization from Monodromy Closure in Time–Scalar Field Theory," *ZJUP*, 2025.

[4] J. G. Farrell, "Heisenberg-Type Uncertainty from Exact Weyl Structure in TSFT," *ZJUP*, 2026.

[5] J. G. Farrell, "Dirac Spinor Emergence from First-Order Factorization in TSFT," *ZJUP*, 2026.

[6] J. G. Farrell, "Schrödinger Dynamics from the Low-Spectrum Limit of TSFT," *ZJUP*, 2026.

[7] J. G. Farrell, "Born Rule from Additive Projector Measures in Time–Scalar Field Theory," *ZJUP*, 2026.

[8] J. G. Farrell, "A Spectral Mechanism for Hierarchical Mass Emergence in Time–Scalar Field Theory," *ZJUP*, 2026.

[9] J. G. Farrell, "Rivets as Microphysical Phase Anchors in the Temporal Coherence Principle," *ZJUP*, 2026.

[10] A. M. Gleason, "Measures on the Closed Subspaces of a Hilbert Space," *Journal of Mathematics and Mechanics*, vol. 6, pp. 885–893, 1957.

[11] M. H. Stone, "On One-Parameter Unitary Groups in Hilbert Space," *Annals of Mathematics*, vol. 33, no. 3, pp. 643–648, 1932.

[12] M. Reed and B. Simon, *Methods of Modern Mathematical Physics, Vol. I: Functional Analysis.* Academic Press, 1980.

[13] B. C. Hall, *Quantum Theory for Mathematicians.* Springer, 2013.

[14] J. J. Sakurai and J. Napolitano, *Modern Quantum Mechanics*, 2nd ed. Cambridge University Press, 2017.

# A Derivation of the Rivet Mass Scaling

This appendix provides a formal justification for the square-root dependence

$$m_{n,\alpha} \propto \sqrt{\lambda_n},$$

used in the main text. The goal is to show that this scaling is not ad hoc but follows naturally from the first-order (Dirac-type) structure of the TSFT dynamics together with the interpretation of rivets as localized trapped-momentum excitations.

## A.1 Quadratic spectral generator

In the TSFT spectral construction, the scale-chain operator $\mathcal{L}$ is self-adjoint and admits the eigenvalue problem

$$\mathcal{L}\psi = \lambda\psi.$$

At the quadratic level, $\lambda$ governs the stiffness of the mode and plays the role analogous to a squared momentum scale in the effective low-energy description.

We now show how the physically relevant excitation scale becomes proportional to $\sqrt{\lambda}$ once the first-order closure is imposed.

### A.2 First-order (Dirac-type) factorization

As established in the TSFT Dirac-sector analysis, the quadratic generator $\mathcal{L}$ admits a first-order factorization of the schematic form

$$\mathcal{L} \approx \mathcal{D}^{\dagger}\mathcal{D},$$

where $\mathcal{D}$ is a first-order (Dirac-type) operator acting on an appropriate spinor extension of $\mathcal{H}$. The precise Clifford structure is fixed by the requirement that cross terms cancel, yielding the familiar Pauli/Clifford closure conditions.

Let $\psi$ be an eigenmode of $\mathcal{L}$ with eigenvalue $\lambda$. Then, on the positive-energy branch, the factorization implies that the effective excitation scale $\mu$ associated with the first-order dynamics satisfies

$$\mu^2 \sim \lambda.$$

Equivalently,

$$\mu \sim \sqrt{\lambda}.$$

### A.3 Physical excitation scale

In the TSFT interpretation, rivets correspond to localized, phase-locked excitations whose inertial response is governed by the first-order effective dynamics. The physically relevant energy (and therefore rest-mass scale in natural units) is thus controlled by the linear excitation scale $\mu$, rather than directly by the quadratic stiffness $\lambda$.

Consequently, the natural TSFT identification is

$$m \propto \mu \propto \sqrt{\lambda}.$$

This establishes the square-root dependence used in the main mass dictionary.

### A.4 Rivet interpretation and normalization

Within the Temporal Coherence Principle (TCP) framework, a rivet represents a localized phase-locked excitation of the underlying TSFT field. Such an excitation behaves as a trapped momentum structure whose inertial response is governed by the effective first-order dynamics described above.

Because the quadratic operator $\mathcal{L}$ controls the stiffness of the mode while the physical excitation scale is linear in the first-order generator, the rest-mass associated with a TCP-stable rivet must scale with the linear excitation scale. Therefore the proportionality

$$m_{n,\alpha} = K\,\sqrt{\lambda_n}\,G(\alpha)$$

is the minimal identification consistent with:

1. the self-adjoint spectral structure of $\mathcal{L}$,
2. the first-order (Dirac-type) closure of the TSFT dynamics,
3. and the interpretation of particles as localized, persistent rivet excitations.

### A.5 Remarks on alternatives

One could formally consider a quadratic mapping $m^2 \propto \lambda$; however, such a choice would correspond to treating the quadratic generator itself as the physical energy operator, bypassing the first-order structure forced by the TSFT Dirac sector. The square-root relation is therefore the natural and structurally preferred identification within the present framework.

### A.6 Conclusion of the appendix

We conclude that the square-root scaling used in the main text is not an adjustable modeling choice but follows directly from the interplay between TSFT spectral stiffness, first-order factorization, and TCP-stable rivet dynamics.

## B Formal Parameter Count and Identifiability

In this appendix we make precise the parameter economy of the TSFT rivet mass relation. The goal is to show that the mapping

$$m_{n,\alpha} = K \sqrt{\lambda_n}\, G(\alpha)$$

contains only a single continuous calibration parameter, with all remaining structure fixed by the TSFT spectral construction and discrete sector choices.

### B.1 Parameter space definition

Let $\mathcal{P}$ denote the space of quantities entering the mass map. From the main text these consist of:

- the global scale $K$,
- the admissible eigenvalues $\{\lambda_n\}$,
- the holonomy sector label $\alpha$,
- structural TSFT constants (e.g. $\phi$, Dirac-sector coefficients).

We now classify which of these represent genuine adjustable degrees of freedom.

### B.2 Continuous versus structural quantities

We distinguish between true continuous calibration parameters and quantities fixed by the TSFT construction.

**Global scale.** The constant $K$ converts TSFT spectral units to physical mass units. It is not determined internally by the present formalism and must be fixed by comparison with a single experimental reference mass. Thus $K$ represents one genuine continuous degree of freedom.

**Admissible spectrum.** The eigenvalues $\{\lambda_n\}$ are not free parameters. They are determined by:

1. the self-adjoint scale-chain operator,
2. monodromy closure,
3. boundary or half-line admissibility.

Once the TSFT operator and admissibility class are specified, the set $\{\lambda_n\}$ is fixed.

**Holonomy sector.** The label $\alpha$ enters as a discrete sector identifier associated with the SU($N$)-covariant structure. Within the present framework it is not treated as a continuously tunable fit parameter but as a structural sector choice.

**Geometric and algebraic constants.** Quantities such as the scale ratio $\phi$ and the coefficients entering the Dirac-type closure arise from the TSFT operator geometry and Clifford consistency conditions. They are fixed once the underlying construction is specified.

### B.3 Counting of continuous degrees of freedom

Under these assumptions, the mass relation depends on exactly one continuous parameter, namely $K$. All remaining dependence is through either:

- spectrally determined eigenvalues $\lambda_n$, or
- discrete sector labels $\alpha$.

### B.4 Identifiability of the mass hierarchy

Because the admissible spectrum $\{\lambda_n\}$ is fixed by the TSFT operator together with the monodromy and boundary conditions, and because the holonomy label $\alpha$ enters only as a discrete sector identifier, the calibrated mass ratios take the form

$$\frac{m_{n,\alpha}}{m_{n_0,\alpha_0}} = \frac{\sqrt{\lambda_n}\, G(\alpha)}{\sqrt{\lambda_{n_0}}\, G(\alpha_0)}.$$

This expression contains no continuous freedom once the global scale $K$ has been fixed by a single reference mass.

### B.5 Predictive statement

We therefore obtain the following structural result.

**Proposition 1** (Single-parameter mass hierarchy)**.** *Within the TSFT rivet framework described in the main text, the spectrum of particle mass ratios is completely determined by the admissible TSFT eigenvalues and holonomy sectors after one global calibration of $K$.*

*Proof.* The mass map depends continuously only on $K$. The remaining quantities entering the ratio above are either spectrally determined ($\lambda_n$) or discrete ($\alpha$). Upon fixing $K$ using a single reference mass, it cancels from all ratios, leaving a fully determined dimensionless hierarchy. □

### B.6 Interpretive remark

The significance of the result is not that the numerical values of the spectrum have already been computed, but that the TSFT framework reduces the mass-hierarchy problem to a well-posed spectral calculation with a single continuous calibration parameter. This provides a clear pathway for quantitative falsification once the admissible spectrum is evaluated explicitly.

**Program status and next steps.** The present work completes the structural bridge between the TSFT spectral-geometry framework and a parameter-economical mechanism for particle mass generation. By formalizing TCP-stable rivet modes and establishing the calibrated mass dictionary

$$m_{n,\alpha} = K\sqrt{\lambda_n}\,G(\alpha),$$

we have reduced the mass-hierarchy problem to a well-posed spectral determination with a single continuous calibration parameter. The remaining task is therefore quantitative rather than structural.

In forthcoming work we will perform explicit numerical evaluation of the admissible TSFT spectrum and the associated holonomy sectors, producing the first concrete mass-ratio tables implied by the rivet mechanism. Those results will provide the primary phenomenological test of whether the TSFT spectral hierarchy aligns with the observed pattern of elementary particle masses and will determine the empirical viability of the framework.