Emergent Newtonian Gravity from Scalar Potential Geometry Without a Fundamental Coupling Constant

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Abstract

The gravitational constant G is traditionally treated as a fundamental parameter in both Newtonian gravity and General Relativity, yet its physical origin and numerical value are not derivable from deeper dynamical principles. In this work, we present an alternative geometrical derivation of gravitational acceleration in which inverse–square behavior arises from spatial gradients of a scalar potential field associated with local time–rate structure, without introducing a fundamental coupling constant. Gravitational acceleration emerges directly as $\boldsymbol{a} = -\nabla\Theta$, where $\Theta(\boldsymbol{x})$ represents a scalar temporal potential whose spatial curvature governs motion. Under spherical symmetry and weak–gradient conditions, solutions to Laplace's equation yield inverse–square scaling as a purely geometric consequence of field propagation. Within this formulation, the quantity historically written as GM appears only as an effective scaling parameter associated with source normalization and unit calibration, not as a fundamental interaction strength. This approach provides a conceptually transparent pathway for introducing gravitational geometry prior to tensor curvature formalism and offers pedagogical continuity between Newtonian and relativistic interpretations of gravity.

1 Introduction

In Newtonian mechanics, gravitational acceleration is written as

$$\boldsymbol{a} = -G\frac{M}{r^2}\,\hat{\boldsymbol{r}}, \tag{1}$$

where G is introduced as a universal coupling constant whose numerical value is determined empirically. In General Relativity, while gravitational phenomena are described as curvature of spacetime geometry, Einstein's field equations still require G as a proportionality constant relating matter–energy to curvature. No known symmetry principle or microscopic field theory predicts the magnitude of G, nor explains why it assumes its observed value.

From a pedagogical perspective, this presents a conceptual difficulty. Students encounter gravity as either an inverse–square force law with an unexplained constant, or as geometric curvature governed by tensor equations that still contain an unexplained constant. In both cases, the interaction strength appears externally imposed rather than emergent from field geometry itself.

In this paper, we present a geometrical derivation of gravitational acceleration based on spatial gradients of a scalar potential field representing local time–rate structure. Within this framework, acceleration follows directly from potential geometry without introducing a fundamental coupling constant. Inverse–square behavior emerges from Laplacian field solutions under spherical symmetry. The constant traditionally labeled G appears only as a macroscopic scaling factor relating observational units to underlying field normalization.

The purpose of this work is not to replace General Relativity, but to provide a pedagogically useful geometric interpretation of Newtonian gravity that naturally anticipates geodesic motion without invoking force mediation or fundamental coupling constants. The intent of this formulation is not to introduce new gravitational dynamics, but to reframe standard Newtonian gravity in geometric terms that emphasize field topology rather than force mediation. All empirical predictions of classical gravity are preserved.

The contribution here is pedagogical: inverse–square behavior is derived from Laplacian geometry under symmetry, and gravitational acceleration is presented as kinematic response to potential gradients, providing conceptual continuity with geodesic motion in weak–field relativity.

2 Scalar Potential as Local Time–Rate Field

Let $\Theta(\boldsymbol{x})$ denote a scalar field representing local time–rate structure. Regions of uniform Θ correspond to inertial frames, while spatial variation in Θ corresponds to differential temporal flow experienced by matter.

Dimensionally, Θ carries units of velocity squared, consistent with its identification with Φ/c^2 in the weak–field relativistic limit, while ρ_R has corresponding source-density units under Laplacian normalization. The specific mapping between ρ_R and conventional mass density depends on the chosen normalization convention and does not affect the functional form of the resulting acceleration field.

We define the gravitational acceleration experienced by matter as the spatial gradient of this scalar field:

$$\boldsymbol{a} = -\nabla\Theta. \tag{2}$$

Spatial curvature of Θ arises from matter distributions that locally impede temporal propagation. We write the field equation as

$$\nabla^2\Theta = \rho_R(\boldsymbol{x}), \tag{3}$$

where ρ_R represents a source density (interpretable as "temporal resistance density") associated with matter. This equation plays the same structural role as Poisson's equation in Newtonian gravity, but the source term is interpreted geometrically rather than as mass density multiplied by a coupling constant.

3 Variational Derivation of Particle Motion

Particle trajectories may be derived from the Lagrangian

$$L = \frac{1}{2}m\dot{\boldsymbol{x}}^2 + m\Theta(\boldsymbol{x}), \tag{4}$$

where the second term represents coupling to the scalar temporal potential.

Applying the Euler–Lagrange equation,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\boldsymbol{x}}}\right) - \frac{\partial L}{\partial \boldsymbol{x}} = 0, \tag{5}$$

yields

$$m\ddot{\boldsymbol{x}} = -m\nabla\Theta. \tag{6}$$

The mass cancels explicitly, giving

$$\boldsymbol{a} = -\nabla\Theta, \tag{7}$$

demonstrating that inertial and gravitational responses arise from the same geometric coupling. This provides a direct route to the equivalence of inertial and gravitational mass in this potential formulation. Θ carries units of velocity squared, consistent with gravitational potential energy per unit mass. This ensures consistency of the Lagrangian term $m\Theta$ with energy units.

4 Spherically Symmetric Field and Inverse–Square Law

Consider a static, spherically symmetric distribution of source strength. Outside the source region, $\rho_R(\boldsymbol{x}) = 0$, and the scalar field satisfies Laplace's equation:

$$\nabla^2\Theta = 0. \tag{8}$$

In three dimensions, the radially symmetric solution is

$$\Theta(r) = \Theta_0 + \frac{C}{r}, \tag{9}$$

where C is determined by boundary conditions associated with the integrated source strength.

Taking the gradient yields the acceleration:

$$\boldsymbol{a}(r) = -\nabla\Theta = -\frac{C}{r^2}\,\hat{\boldsymbol{r}}. \tag{10}$$

Thus inverse–square behavior emerges as a geometric consequence of Laplacian field propagation under spherical symmetry. No inverse–square force law is imposed; the scaling follows from the structure of solutions to $\nabla^2\Theta = 0$.

5 Interpretation of the Gravitational Constant

In classical notation, gravitational acceleration is written

$$\boldsymbol{a} = -G\frac{M}{r^2}\,\hat{\boldsymbol{r}}. \tag{11}$$

Comparing with the scalar–potential result,

$$\boldsymbol{a} = -\frac{C}{r^2}\,\hat{\boldsymbol{r}}, \tag{12}$$

we identify

$$C \equiv GM. \tag{13}$$

Within the present framework, however, C arises from integration of the source term. For a compact source,

$$C = \kappa_\Theta \int \rho_R(\boldsymbol{x})\, dV, \tag{14}$$

where κ_Θ collects normalization factors associated with the field response and the calibration between observational units and the underlying potential normalization.

In this sense, G functions as an effective scaling parameter in the mapping from a chosen source normalization to measured accelerations, rather than as a fundamental coupling constant required by the acceleration law itself. This geometric relation is illustrated in Fig. 1

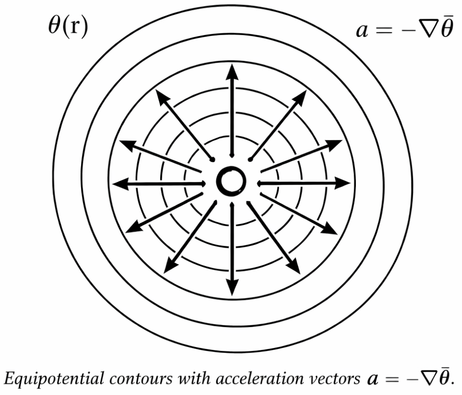

Figure 1: Illustration of scalar potential contours $\Theta(r)$ under spherical symmetry. Acceleration vectors $\boldsymbol{a} = -\nabla\Theta$ point orthogonally to equipotential surfaces and increase in magnitude where contour spacing decreases, producing inverse–square scaling.

6 Relation to Weak–Field Relativity

In weak–field General Relativity, one often writes

$$g_{00} \approx 1 + \frac{2\Phi}{c^2}, \tag{15}$$

where Φ is the Newtonian gravitational potential. Identifying

$$\Theta \equiv \frac{\Phi}{c^2}, \tag{16}$$

yields the approximate line element

$$ds^2 \approx (1 + 2\Theta)\, c^2 dt^2 - d\boldsymbol{x}^2. \tag{17}$$

In this limit, the geodesic equation reduces to

$$\frac{d^2\boldsymbol{x}}{dt^2} = -\nabla\Theta, \tag{18}$$

matching the scalar–potential acceleration law. This provides a conceptually transparent bridge: weak–field geodesic motion corresponds to kinematic response to gradients in an effective temporal potential.

7 Pedagogical Implications

This formulation offers several advantages for instructional contexts:

- Gravitational acceleration is introduced directly as motion along potential gradients, consistent with prior exposure to conservative fields.

- Inverse–square behavior is derived from Laplacian geometry under symmetry rather than postulated as a force law.
- The equivalence of inertial and gravitational mass follows immediately from the variational derivation.
- Continuity between Newtonian gravity and weak–field relativistic geodesics is conceptually clear.
- The gravitational constant appears as a scaling artifact tied to normalization and unit calibration rather than a required fundamental coupling.

For practical calibration, the constant C may be fixed by matching a known reference acceleration. For example, at Earth's surface one requires $a(R_\oplus) = 9.81\ \mathrm{m/s}^2$, which yields $C = aR_\oplus^2$, numerically equivalent to the classical value $GM_\oplus$. Thus standard gravitational predictions are recovered exactly, while the acceleration law itself remains independent of an assumed coupling constant.

8 Conclusion

We have presented a geometrical derivation of Newtonian gravitational acceleration based on spatial gradients of a scalar temporal potential field. Inverse–square behavior arises naturally from Laplacian field solutions under spherical symmetry, without introducing a fundamental coupling constant. The gravitational constant appears only as an effective scaling parameter associated with source normalization and unit calibration.

This formulation preserves empirical equivalence with classical gravity while providing a unified interpretation of inertial and gravitational motion as kinematic response to field geometry. It offers a pedagogically accessible bridge between Newtonian potential theory and weak–field relativistic geodesic motion, emphasizing geometry over force mediation.

Author Declarations

Conflict of Interest

The author declares no conflicts of interest.

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