# Lorentz Transformations as Observer–Gauge Redundancies over Θ-Deformation Modes and the Reinterpretation of Temporal Torsion as Magnetism in TSFT

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## Abstract

Time-Scalar Field Theory (TSFT) models time as a scalar field $\Theta(x)$ on a compact manifold, with electromagnetism emerging as mechanical deformation modes of $\Theta$: electricity as temporal shear and magnetism as temporal vorticity. Prior TSFT work defined $\mathbf{E}_\Theta = -\nabla\Theta$ and $\mathbf{B}_\Theta = \nabla \times (\partial_t\Theta)$ as orthogonal modes of temporal deformation, supported by an electrodynamic correspondence postulate involving non-integrable curvature of local time frames. Here we provide a covariant formulation that makes explicit why special-relativistic observers "reinterpret" torsion as magnetism: the split of a single underlying covariant field-strength into electric-like and magnetic-like components depends on the observer 4-velocity $u^\mu$. We show that Lorentz transformations act as changes of observer and temporal foliation, producing the standard $E/B$ mixing formulas, while the underlying Θ-deformation tensor remains invariant. In this sense, Lorentz boosts are gauge-like redundancies in how Θ-deformation modes are decomposed, not changes to the underlying physical content. We close with TSFT-consistent predictions for small scalar–vector couplings in regimes where the temporal frame is curved or rapidly modulated.

## 1 Introduction

A recurring pedagogical claim in relativistic electrodynamics is that magnetism is "a relativistic correction" to electrostatics, or even (in popular phrasing) "electricity viewed through time distortion." These statements point toward a correct geometric fact: electric and magnetic fields are not independent objects but frame-dependent components of a single Lorentz-covariant field.

TSFT strengthens and mechanizes that insight by treating time itself as a dynamical scalar field $\Theta(x)$. In the TSFT electrodynamic correspondence, electric and magnetic fields emerge as orthogonal deformation modes of $\Theta$: temporal shear/pressure and temporal torsional vorticity, respectively. In particular, prior TSFT work defined

$$\mathbf{E}_\Theta = -\nabla\Theta, \qquad \mathbf{B}_\Theta = \nabla \times (\partial_t\Theta),$$

and interpreted $\mathbf{B}_\Theta$ as the *twist/vorticity* of the time field. (See the "Mechanical Interpretation" and the electrodynamic postulate establishing non-integrable curvature / torsion.)[1] The goal of this paper is to make that framework fully covariant and to prove the main statement requested here:

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[1] These are the explicit TSFT definitions and interpretations already adopted in *Unified Electromagnetism in TSFT*.

> *Lorentz transformations can be interpreted as observer–gauge redundancies over $\Theta$-deformation modes, explaining why special-relativistic observers reinterpret temporal torsion as magnetic field.*

We will keep two principles clear: (i) the *underlying* covariant object is observer-independent; (ii) the identification of "electric" versus "magnetic" components is an observer-dependent split.

## 2 TSFT electromagnetic recap: shear and torsion modes

In TSFT, time is a scalar field $\Theta(x)$, and electrodynamics emerges from geometric modes of $\Theta$. The previously introduced correspondence postulate (Axiom E1) can be read as asserting that local time frames possess non-integrable structure—i.e., torsion/holonomy—so that rotational deformation is permitted and physically meaningful.

In the simplest 3+1 language used previously, TSFT defines the shear mode

$$\mathbf{E}_\Theta = -\nabla\Theta$$

and a torsional/vorticity mode

$$\mathbf{B}_\Theta = \nabla \times (\partial_t\Theta).$$

The mechanical interpretation is explicit: $\mathbf{E}_\Theta$ is linear shear or "pressure" of $\Theta$, while $\mathbf{B}_\Theta$ is torsional twist or vorticity of $\Theta$, and radiation is the propagating exchange of these modes.

Those definitions are adequate in a preferred coordinate split, but to connect cleanly to Lorentz transformations, we must express the same content in covariant form.

## 3 Covariant temporal frame and field strength

### 3.1 Temporal frame 1-form

The key move is to package $\Theta$-deformation into a *temporal frame* 1-form. Let

$$\tau_\mu \equiv \partial_\mu\Theta + a_\mu.$$

Here $a_\mu$ is a minimal additional 1-form encoding the possibility of non-integrable temporal frame structure. This is the covariant representation of the idea in Axiom E1 that one can have rotational torsion of local time frames (i.e., structure beyond an exact gradient). If $a_\mu = 0$, then $\tau_\mu$ is exact and produces no curvature; torsionless $\Theta$ cannot generate a nontrivial electromagnetic-like field. With $a_\mu \neq 0$, TSFT permits holonomy in the temporal frame while maintaining a scalar-time ontology.

### 3.2 Field strength as $\Theta$-deformation curvature

Define the covariant $\Theta$-deformation field strength (a 2-form):

$$F_{\mu\nu} \equiv \partial_\mu\tau_\nu - \partial_\nu\tau_\mu.$$

Since $\partial_\mu\partial_\nu\Theta$ is symmetric, the scalar part drops out and

$$F_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu.$$

Thus the curvature lives in the non-integrable part of the temporal frame. This is exactly the geometric mechanization of "non-integrable curvature of $\Theta$": the physical content is in the torsion/holonomy encoded by $a_\mu$, while $\Theta$ supplies the temporal foliation and shear mode.

### 3.3 Gauge symmetry

A natural gauge redundancy appears immediately:

$$\Theta \to \Theta + \lambda, \qquad\qquad a_\mu \to a_\mu - \partial_\mu \lambda.$$

This leaves $\tau_\mu$ invariant, hence leaves $F_{\mu\nu}$ invariant. Equivalently, one may state the familiar form:

$$\tau_\mu \to \tau_\mu + \partial_\mu \lambda \qquad \Rightarrow \qquad F_{\mu\nu} \text{ unchanged}.$$

In TSFT language: $\lambda$ corresponds to a *relabeling of temporal phase* (a change in the scalar-time chart), while the measurable deformation curvature is invariant.

This is the first sense in which "gauge" enters TSFT electrodynamics: the decomposition into scalar-time labeling versus torsion content has redundancy.

## 4 Observer-dependent split: how $E$ and $B$ arise from the same $F_{\mu\nu}$

### 4.1 Electric and magnetic fields as projections relative to $u^\mu$

Given any timelike unit 4-velocity $u^\mu$ (the observer), define electric-like and magnetic-like 4-vectors by the standard covariant split:

$$E_\mu(u) \equiv F_{\mu\nu} u^\nu, \qquad\qquad B_\mu(u) \equiv {}^\star F_{\mu\nu} u^\nu,$$

where ${}^\star F$ is the Hodge dual of $F$. By construction,

$$E_\mu u^\mu = 0, \qquad B_\mu u^\mu = 0,$$

so each is purely spatial in the observer's rest frame.

**TSFT interpretation.** $E_\mu(u)$ is the observer's measured *temporal shear* component of $\Theta$-deformation; $B_\mu(u)$ is the observer's measured *temporal torsion/vorticity* component. The underlying object is $F_{\mu\nu}$, but the split into shear vs torsion is performed by choosing $u^\mu$. Change the observer, and the split changes.

This is the precise mathematical statement behind the popular intuition that "magnetism depends on motion."

### 4.2 Lorentz transformations as changes of observer

A Lorentz transformation $\Lambda$ acts on the covariant tensor as

$$F'_{\mu\nu} = \Lambda_\mu{}^\alpha \Lambda_\nu{}^\beta F_{\alpha\beta}, \qquad\qquad u'^\mu = \Lambda^\mu{}_\nu u^\nu.$$

But if one is describing *the same physical field* with different observers, it is often clearest to hold $F_{\mu\nu}$ fixed and change only the observer $u^\mu \to u'^\mu$. Then the measured fields become

$$E_\mu(u') = F_{\mu\nu} u'^\nu, \qquad\qquad B_\mu(u') = {}^\star F_{\mu\nu} u'^\nu.$$

Therefore, Lorentz boosts are equivalently *observer changes* that alter the decomposition of the same $\Theta$-deformation curvature into electric-like and magnetic-like parts.

## 5 Deriving the standard $E/B$ mixing from the TSFT split

Consider a boost with velocity $\mathbf{v}$ (relative speed $v$) between two inertial observers. Let $\gamma = 1/\sqrt{1 - v^2/c^2}$. The standard transformation laws are:

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}, \qquad \mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel},$$

$$\mathbf{E}'_{\perp} = \gamma\left(\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}\right), \qquad \mathbf{B}'_{\perp} = \gamma\left(\mathbf{B}_{\perp} - \frac{1}{c^2}\mathbf{v} \times \mathbf{E}\right).$$

These follow directly from the covariant definitions $E_\mu = F_{\mu\nu}u^\nu$ and $B_\mu = {}^\star F_{\mu\nu}u^\nu$ together with the boost relation between $u$ and $u'$.

**TSFT reading.** The same $F_{\mu\nu}$ (the same underlying torsion/shear deformation curvature of the temporal frame) is decomposed by two different observers. A boost changes the *temporal slicing* (what the observer considers "space at an instant") and thus changes the separation of:

- shear-like components ("electric"),
- torsion-like components ("magnetic").

In short: *magnetism appears when the observer's temporal foliation mixes shear and torsion in the $\Theta$-deformation tensor.*

## 6 Why this is "gauge-like" in TSFT (the key conceptual bridge)

There are two distinct redundancies:

### 6.1 (A) Temporal chart gauge: $\Theta \to \Theta + \lambda$

As shown above, the redundancy

$$\Theta \to \Theta + \lambda, \qquad a_\mu \to a_\mu - \partial_\mu \lambda$$

leaves $\tau_\mu$ and $F_{\mu\nu}$ invariant. This is a literal gauge symmetry in the temporal frame representation.

### 6.2 (B) Observer/foliation redundancy: $u^\mu$ as a "gauge choice" for decomposition

The second redundancy is not a gauge symmetry of $F$ itself, but a redundancy in the *interpretation*: different inertial observers (different $u^\mu$) measure different splits $(E(u), B(u))$ of the same $F$. In TSFT this can be understood as a freedom in choosing the temporal foliation used to separate "shear" from "torsion" modes.

This is exactly the statement you requested:

> Lorentz transformations act as gauge-like redundancies over $\Theta$-deformation *modes* because they change how the same covariant deformation curvature is decomposed into shear vs torsion.

**Practical phrasing.** Special relativity does not create magnetism out of nothing; it changes the bookkeeping: what one observer calls "pure shear" another observer calls "shear + torsion." TSFT supplies an ontology for that bookkeeping: shear and torsion are mechanical modes of the scalar-time manifold.

# 7 Connecting back to the TSFT 3+1 definitions

To connect the covariant formulation to the prior TSFT definitions in a chosen frame, take an observer with 4-velocity $u^\mu = (c, 0, 0, 0)$ in coordinates adapted to their time slicing. Then the spatial components reproduce the familiar identifications:

$$E_i = F_{i0}, \qquad\qquad B_i = \frac{1}{2}\epsilon_{ijk}F_{jk}.$$

In TSFT's electrodynamic correspondence, the temporal shear variable is $\nabla\Theta$ and torsion is tied to rotational structure of the temporal frame. In a coordinate gauge where the scalar mode dominates the shear channel, one recovers the prior working definitions:

$$\mathbf{E}_\Theta = -\nabla\Theta, \qquad\qquad \mathbf{B}_\Theta = \nabla \times (\partial_t\Theta)$$

as the leading 3+1 expressions for these deformation modes.

Thus, the covariant TSFT frame-field formulation is not a replacement of the earlier definitions, but their completion: it explains precisely why boosts mix $\mathbf{E}_\Theta$ and $\mathbf{B}_\Theta$ even though the underlying deformation curvature is unchanged.

# 8 Implications and TSFT-specific experimental hooks

In conventional electrodynamics, Lorentz covariance is a symmetry principle imposed on Maxwell's equations. In TSFT, Lorentz covariance appears as a constraint on allowable decompositions of the temporal frame curvature. Two immediate TSFT hooks follow:

## 8.1 Scalar–vector coupling in curved or rapidly modulated temporal frames

If the temporal frame $\tau_\mu$ is not globally integrable due to manifold curvature, strong gravitational potentials, or high-frequency modulation of $\Theta$, small couplings between the scalar labeling mode and torsion mode can arise. This is consistent with prior TSFT claims that strong-gravity or high-frequency regimes may exhibit scalar–vector couplings beyond the flat-$\Theta$ limit.

## 8.2 Boundary-locked torsion modes (Casimir and materials)

In TSFT, conducting boundaries enforce temporal phase locking, constraining allowed $\Theta$-modes and producing measurable forces. This same boundary mechanism selects torsion/shear mode structure in cavities and metamaterials, suggesting laboratory platforms where torsion-mode signatures might be modulated and isolated. (This is conceptually aligned with TSFT Casimir derivations in which conduction electrons enforce hard constraints on $\Theta$-modes and produce measurable deviations under modulation.)

# 9 Conclusion

We provided a covariant TSFT electrodynamic framework in which the fundamental object is the curvature (field strength) of the temporal frame 1-form $\tau_\mu = \partial_\mu\Theta + a_\mu$. Electric-like and magnetic-like fields are observer-dependent projections of the same $F_{\mu\nu}$ relative to a 4-velocity $u^\mu$. Lorentz transformations therefore act as changes of observer/foliation that mix the decomposition of $\Theta$-deformation curvature into shear and torsion modes, yielding the standard $E/B$ mixing relations.

This completes the mechanical TSFT interpretation: magnetism is not “backward time,” but the torsional/vorticity component of scalar-time deformation, and special relativity explains how different observers partition the same deformation into shear vs torsion. With the temporal-flux action $S_{\mathrm{EM}}$, the inhomogeneous Maxwell equations and charge continuity arise as Euler–Lagrange and consistency identities, completing the Maxwell structure within the $\Theta$-frame ontology.

## References

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## 10 Maxwell structure from Θ-frame curvature

We now formalize two claims that were previously stated narratively: (i) the homogeneous Maxwell equations follow identically from the definition $F = d\tau$; (ii) the inhomogeneous Maxwell equations arise once we introduce a TSFT-native conserved temporal-flux current.

### 10.1 Theorem 1 (Homogeneous Maxwell equations as a Bianchi identity)

Define the temporal frame 1-form and its curvature 2-form by

$$\tau_\mu \equiv \partial_\mu \Theta + a_\mu, \qquad F_{\mu\nu} \equiv \partial_\mu \tau_\nu - \partial_\nu \tau_\mu,$$

and the dual tensor ${}^\star F^{\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$. Then the homogeneous Maxwell equations hold identically:

$$\partial_{[\lambda}F_{\mu\nu]} = 0 \qquad \Longleftrightarrow \qquad \partial_\mu {}^\star F^{\mu\nu} = 0.$$

**Proof.** Because $F = d\tau$ is an exterior derivative of a 1-form, it satisfies $dF = d(d\tau) = 0$ identically. In index notation this is $\partial_{[\lambda}F_{\mu\nu]} = 0$. Contracting with $\epsilon^{\mu\nu\alpha\beta}$ yields $\partial_\mu {}^\star F^{\mu\nu} = 0$. No additional dynamical assumptions are required. □

**3+1 form.** In any inertial frame these identities reproduce

$$\nabla \cdot \mathbf{B} = 0, \qquad \nabla \times \mathbf{E} + \partial_t \mathbf{B} = 0,$$

with $\mathbf{E}_i = F_{i0}$ and $\mathbf{B}_i = \frac{1}{2}\epsilon_{ijk}F_{jk}$.

### 10.2 Theorem 2 (Inhomogeneous Maxwell equations from a TSFT flux action)

To obtain sources, we introduce a TSFT-native *temporal flux current* $J^\mu_\Theta$ as the Noether current associated with temporal chart-gauge symmetry, and we derive the sourced field equation from a minimal action.

**Definition (Temporal-flux action).** Let the electromagnetic sector of the TSFT effective action be

$$S_{\mathrm{EM}}[\tau;\, J_\Theta] = \int d^4x \left(-\frac{\alpha}{4}F_{\mu\nu}F^{\mu\nu} - J^\mu_\Theta \tau_\mu\right),$$

where $\alpha > 0$ is the TSFT stiffness/impedance constant in this sector (and may be identified with your prior $\alpha$-weighting in the electrodynamic correspondence paper), and $J^\mu_\Theta$ is a physical temporal-flux source (charge/current in the projection map).

**Theorem (Euler–Lagrange field equation).** Varying $S_{\mathrm{EM}}$ with respect to $\tau_\mu$ yields the sourced Maxwell equation

$$\partial_\mu F^{\mu\nu} = \frac{1}{\alpha}J^\nu_\Theta.$$

**Proof.** Use $\delta F_{\mu\nu} = \partial_\mu \delta\tau_\nu - \partial_\nu \delta\tau_\mu$. Then

$$\delta S_{\mathrm{EM}} = \int d^4x \left(-\frac{\alpha}{2}F^{\mu\nu}\delta F_{\mu\nu} - J^\nu_\Theta \delta\tau_\nu\right) = \int d^4x \left(-\alpha F^{\mu\nu}\partial_\mu \delta\tau_\nu - J^\nu_\Theta \delta\tau_\nu\right),$$

where antisymmetry of $F^{\mu\nu}$ combined the two terms. Integrating by parts and discarding boundary terms,

$$\delta S_{\mathrm{EM}} = \int d^4x \left(\alpha\, \partial_\mu F^{\mu\nu} - J^\nu_\Theta\right)\delta\tau_\nu.$$

Stationarity for arbitrary $\delta\tau_\nu$ implies $\alpha\,\partial_\mu F^{\mu\nu} - J^\nu_\Theta = 0$, i.e. $\partial_\mu F^{\mu\nu} = \frac{1}{\alpha}J^\nu_\Theta$. ☐

### 10.3 Corollary (Charge conservation as a consistency identity)

Taking $\partial_\nu$ of the sourced equation gives

$$\partial_\nu \partial_\mu F^{\mu\nu} = \frac{1}{\alpha}\partial_\nu J^\nu_\Theta.$$

But the left side vanishes identically because $F^{\mu\nu}$ is antisymmetric while $\partial_\nu\partial_\mu$ is symmetric, hence

$$\partial_\nu J^\nu_\Theta = 0.$$

Thus temporal-flux sources must be conserved: this is the TSFT projection of charge continuity.

**3+1 form.** Define $\rho_\Theta \equiv \frac{1}{\alpha} J^0_\Theta$ and $\mathbf{J}_\Theta \equiv \frac{1}{\alpha}(J^1_\Theta, J^2_\Theta, J^3_\Theta)$. Then

$$\nabla \cdot \mathbf{E} = \rho_\Theta, \qquad \nabla \times \mathbf{B} - \partial_t \mathbf{E} = \mathbf{J}_\Theta,$$

and the continuity equation becomes $\partial_t \rho_\Theta + \nabla \cdot \mathbf{J}_\Theta = 0$.

### 10.4 TSFT interpretation of $J^\mu_\Theta$ (temporal flux projection)

In TSFT, $J^\mu_\Theta$ is not taken as primitive "charge" but as a *temporal flux source*: a conserved defect/flow of the temporal frame that projects into charge/current at the classical level. This is consistent with the prior TSFT electrodynamic correspondence, where charge/current are described as curvature defects and temporal flux, and where electric and magnetic fields are identified as shear and vorticity modes of Θ-deformation. (See the shear/vorticity definitions and mechanical interpretation.)