A Transdimensional Scalar-Time Derivation of the Casimir Effect: Compatibility, Corrections, and Testable Predictions in TSFT

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Abstract

We present a proof program deriving the Casimir effect within Time-Scalar Field Theory (TSFT), where vacuum fluctuations emerge as constrained modes of a scalar-time coherence field Θ. Starting from TSFT postulates, we recover the standard attractive pressure $P(a) = -\frac{\pi^2 \hbar c}{240 a^4}$ in the ideal limit via mode quantization and renormalization interpreted as coherence-baseline selection. TSFT-specific corrections—a coherence gap inducing exponential suppression at large separations and a modulation signature under external temporal perturbations—yield falsifiable predictions distinguishable from QED. Proposed experiments using AFM and lock-in detection are outlined, positioning TSFT as empirically viable for unified physics.

1 Introduction

Time-Scalar Field Theory (TSFT) posits time as an active scalar field $\Theta(x, t)$ whose gradients encode temporal coherence, unifying quantum, electromagnetic, gravitational, and thermodynamic domains under constraint projections (Transdimensional Identity, TDI). Here, we apply TSFT to the Casimir effect, traditionally viewed as electromagnetic vacuum energy shifts between conducting plates. In TSFT, it manifests as a boundary-induced imbalance in Θ-mode coherence density.

This paper structures as: definitions and postulates (§2), lemmas on mode structure and energy shifts (§3), the core theorem recovering standard Casimir (§4), TSFT corrections with predictions (§5), experimental proposals (§6), and conclusions (§7).

2 Definitions and Postulates

Consider infinite parallel conducting plates of area A separated by distance a along z.

Definition 1 (TSFT Vacuum Functional)**.** *The vacuum partition function is*

$$\mathcal{Z} = \int \mathcal{D}\Theta \, \exp\left(\frac{i}{\hbar}\int d^4x \, \mathcal{L}_\Theta\right), \tag{1}$$

with minimal relativistic Lagrangian

$$\mathcal{L}_\Theta = \frac{\kappa}{2}\partial_\mu\Theta\partial^\mu\Theta - \frac{\kappa\omega_0^2}{2c^2}\Theta^2, \tag{2}$$

where κ is temporal stiffness (unified TSFT constant), and ω_0 a coherence gap (set to 0 initially).

Postulates: 1. Θ gradients define temporal impedance/coherence. 2. Vacuum fluctuations are Θ-constraint responses. 3. Plates impose Dirichlet boundaries: $\Theta(z = 0) = \Theta(z = a) = 0$, reflecting electron-mediated phase locking (per TDI).

3 Lemmas: Mode Structure and Energy Shifts

Lemma 1 (Mode Quantization)**.** *Boundaries yield*

$$\Theta(z = 0) = \Theta(z = a) = 0 \implies k_z = \frac{n\pi}{a}, \; n = 1, 2, \dots \tag{3}$$

Dispersion ($\omega_0 = 0$):

$$\omega_{n,\mathbf{k}_\perp} = c\sqrt{k_\perp^2 + \left(\frac{n\pi}{a}\right)^2}. \tag{4}$$

Lemma 2 (Vacuum Energy Difference)**.** *Cavity energy:*

$$E(a) = \frac{\hbar}{2}\sum_{n=1}^{\infty}\int \frac{A\, d^2k_\perp}{(2\pi)^2}\,\omega_{n,\mathbf{k}_\perp}. \tag{5}$$

Renormalized shift: $\Delta E(a) = E(a) - E(\infty)$, where $E(\infty)$ integrates over k_z.

Lemma 3 (Evaluation via Zeta/Abel-Plana)**.** *Yields*

$$\frac{\Delta E(a)}{A} = -\frac{\pi^2\hbar c}{720a^3}. \tag{6}$$

Pressure:

$$P(a) = -\frac{\partial}{\partial a}\left(\frac{\Delta E}{A}\right) = -\frac{\pi^2\hbar c}{240a^4}. \tag{7}$$

3.1 Transdimensional Identity and Boundary Enforcement

The Transdimensional Identity (TDI) provides the conceptual foundation for why conducting boundaries, particularly those mediated by conduction electrons, are essential to the Casimir effect in TSFT. Under TDI, physical domains such as quantum fluctuations, electromagnetic fields, and mechanical forces are not fundamental but emerge as projections of the underlying scalar-time field Θ, differentiated by boundary constraints and induced maps (as detailed in the Grand Unified framework's quantum emergence and electromagnetism pillars [1], where Θ-curvature yields Schrödinger/Dirac equations and EM arises from Θ-shear/vorticity). In this view, conducting plates do not primarily act as electromagnetic mirrors; instead, they enforce hard boundary conditions on Θ-modes through electron-mediated phase locking.

Conduction electrons are uniquely suited for this role due to their high mobility, delocalized fermionic coherence, and low effective inertia relative to Θ-variation timescales. These properties allow electrons to respond rapidly to Θ-gradients, collectively locking the temporal phase across the boundary and projecting "soft" scalar constraints into "hard" Dirichlet conditions ($\Theta(z = 0) = \Theta(z = a) = 0$). This explains why metals exhibit strong Casimir forces, while dielectrics or insulators (with trapped electrons) yield weaker or imperfect effects: the coherence medium is insufficiently responsive, permitting Θ-mode leakage. Quantum fluctuations and electromagnetic manifestations are thus secondary artifacts of constrained Θ-mode counting, not primary sources.

In TSFT, the Casimir effect is not evidence of electromagnetic vacuum energy, but of constrained scalar-time coherence, with conduction electrons acting as the enforcing boundary medium via the Transdimensional Identity. This interpretation ties to the unified action S [1], where Θ-induced maps (e.g., $E_\Theta = -\partial_t(\nabla\Theta) - \nabla\Phi_\Theta$ for electric shear) naturally incorporate electron dynamics, predicting extensions like tunable boundaries in superconductors or metamaterials.

4 TSFT Casimir Theorem

Theorem 1 (TSFT Casimir Pressure)**.** *In TSFT vacuum, boundary-constrained Θ-modes yield attractive pressure*

$$P(a) = -\frac{\pi^2 \hbar c}{240 a^4} \tag{8}$$

in massless, ideal limit. Force per area: $F/A = |P(a)|$.

Proof: Follows from Lemmas 1–3. TSFT interpretation: Force equalizes Θ-coherence under constraints, with renormalization as baseline choice.

5 TSFT-Specific Corrections and Predictions

Channel A: Coherence Gap ($\omega_0 \neq 0$) Dispersion:

$$\omega_{n,\mathbf{k}_\perp} = c\sqrt{k_\perp^2 + \left(\frac{n\pi}{a}\right)^2 + \frac{\omega_0^2}{c^2}}. \tag{9}$$

Pressure:

$$P(a) \approx -\frac{\pi^2 \hbar c}{240 a^4}\, \Phi\left(\frac{a}{\lambda_\Theta}\right), \quad \lambda_\Theta = \frac{c}{\omega_0}, \tag{10}$$

with $\Phi(x) \to 1$ ($x \ll 1$), $\Phi(x) \sim e^{-2x}$ ($x \gg 1$).

Prediction: Roll-off in $P(a)$ for $a \gtrsim 0.1 - 10\,\mu$m, beyond finite-conductivity effects.

Channel B: Modulation Signature Under external Θ-modulation (e.g., gravitational or metamaterial):

$$P(a,t) = P_0(a)\left[1 + \epsilon \cos(\Omega t)\right], \tag{11}$$

with $\epsilon \ll 1$.

Prediction: Oscillatory force detectable via lock-in at Ω.

6 Proposed Experiments

1. **Static Deviation Tests**: Use AFM sphere-plane setups (e.g., Lamoreaux et al. [18]) to fit $P(a)$ across $0.1 - 10\,\mu$m; isolate ω_0 from roughness/patch data.

2. **Modulation Detection**: Micromachined oscillators with lock-in amps; modulate via piezoelectric strain or gravity gradients. Sensitivity: $\epsilon \sim 10^{-6} - 10^{-4}$.

Datasets: Cite NIST/PTB Casimir archives; propose ZJUP collaboration for metamaterial cavities.

7 Conclusions and Future Work

This TSFT proof recovers Casimir compatibility while yielding testable deviations, advancing unification. Future: Derive $\hbar$ from TSFT temporal efficiency; extend to dynamic Casimir.

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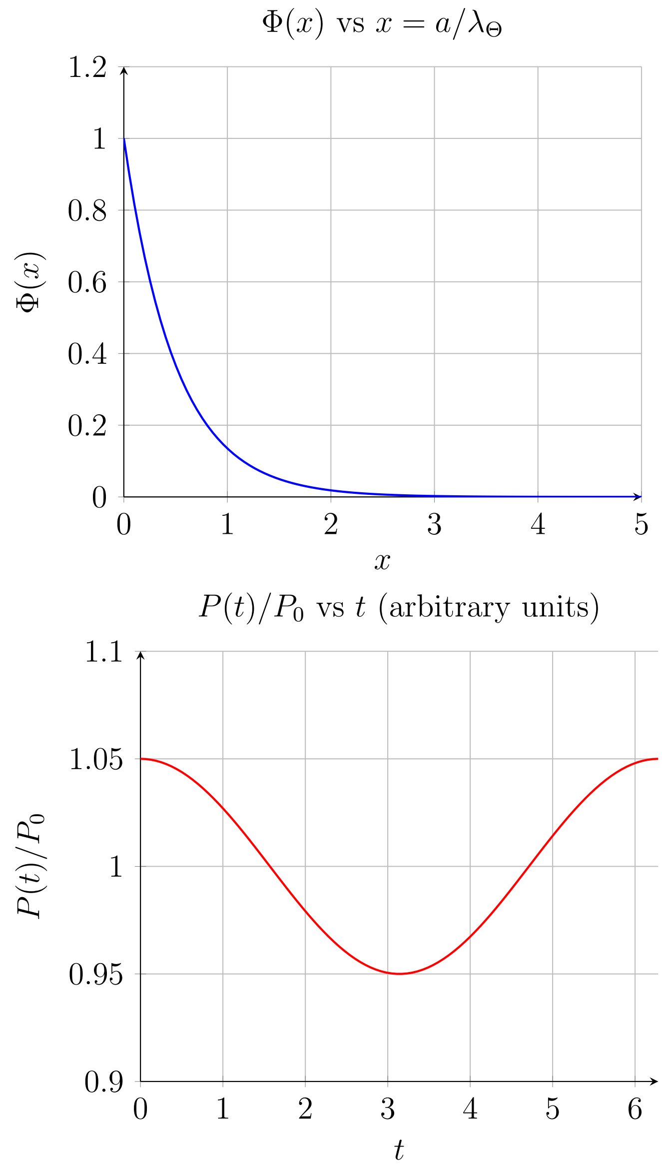

Figure 1: TSFT deviations. Left: The suppression factor $\Phi(x)$, where $x = a/\lambda_\Theta$ and $\lambda_\Theta = c/\omega_0$ is the coherence length from the gap term in the Lagrangian. For small x (separations much smaller than coherence length), $\Phi \approx 1$, matching the ideal limit. As x increases, Φ decreases exponentially $\sim e^{-2x}$, suppressing the force due to fewer admissible Θ-modes. This predicts a roll-off in $P(a)$ beyond QED effects, testable via static measurements. Right: Normalized modulated pressure $P(t)/P_0 = 1+\epsilon\cos(\Omega t)$ (with $\epsilon = 0.05$ for illustration; actual $\epsilon \sim 10^{-6} - 10^{-4}$). This shows the oscillatory signature under external Θ-perturbation, unique to TSFT and detectable via lock-in amplification in dynamic experiments.