Time-Scalar Field Theory Derivation of φ and the Quantum Fractal Bridge

Jordan Gabriel Farrell
512 Springbrook Circle, Colchester, CT 06415
jgfquantum@gmail.com
ORCID: 0009-0002-2171-809X

December 09, 2025

Abstract

We extend Time-Scalar Field Theory (TSFT), as introduced in the seminal work Zebra Poker: The Ultimate Unification of Physics [1], to derive the golden ratio $\varphi = (1+\sqrt{5})/2$ through the Transdimensional Identity. Building on TSFT's phase-closure derivations of π [2] and solar phenomena [3–5], we show that TSFT's scalar-time self-similar flux partition across Θ-layers enforces φ as the invariant growth ratio, yielding bridges to quantum fractal spectra in Bohr, Dirac, and Euler contexts. This unifies algebraic constants with TSFT's conserved energy-flux framework, exposing the fractal nature of quantum mechanics without additional parameters.

1 Introduction

Time-Scalar Field Theory (TSFT) reformulates time as a scalar field $\theta(x)$ on a compact 4-manifold with two boundaries, as detailed in the foundational monograph Zebra Poker: The Ultimate Unification of Physics [1]. Section 2 of [1] introduces the Transdimensional Identity, rooted in self-similar duplications, which enforces flux invariance under layer shifts ($n \to n+1$). Previous TSFT applications have resolved solar coronal heating via second-order curvature $d^2\Theta/dr^2$ [3], wind acceleration via first-order gradients $d\Theta/dr$ [4], Mercury's perihelion precession via an exterior r^{-3} tail [5], and π via elliptic phase-closure [2]. Here, we demonstrate that the same scalar-time machinery derives the golden ratio φ as a byproduct of self-similar flux closure in Θ-layers, emerging as the spectral growth constant compatible with TSFT flux conservation [6, 7]. This matches fractal properties in quantum systems [8–11] and provides a physical anchor for the fractal nature of quantum mechanics.

2 Transdimensional Identity for φ (Zebra Poker Framework)

Drawing from self-similar identities [6, 12], the Transdimensional Identity enforces flux partition across Θ-layers via dimensional duplication.

Theorem 2.1 (TSFT Self-Similar Closure $\Rightarrow \varphi$)**.** Let a Θ-invariant duplication split a quantity S into self-similar parts S_1, S_2 with ratio $r = S_1/S_2$, preserving action. Enforcing invariance under iterative duplication implies $r = 1 + 1/r$. The unique $r > 0$ solution is $\varphi = (1 + \sqrt{5})/2$.

Proof. The quadratic $r^2 - r - 1 = 0$ arises from scale-free Θ-flux partition; the positive root ensures physical growth [6, 13] (see Appendix A for detailed derivation). □

Corollary 2.1.1 (Transfer-Operator Spectrum)**.** For the Θ-layer duplication map

$$T = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix},$$

the dominant eigenvalue is φ. Hence $\lim_{n\to\infty} S_{n+1}/S_n = \varphi$ for any positive initial data.

Proof. Characteristic equation $\det(T - \lambda I) = 0$ yields $\lambda^2 - \lambda - 1 = 0$ [7, 14]. □

Corollary 2.1.2 (Most-Irrational Rotation & KAM Robustness)**.** The continued fraction with all 1's yields $\rho = \varphi^{-1}$. A Θ-phase lattice with rotation ρ maximizes resistance to resonance (minimal mode-locking), producing hierarchical (Fibonacci) fractal spectra upon quantization.

Proof. φ^{-1} is the worst approximable irrational by continued fraction convergents [15, 16]. □

3 Bridges to Bohr/Dirac/Euler and Quantum Fractals

Lemma 3.0.1 (π–φ Angle Identity in Θ-Closure)**.** Five-fold phase closure gives $\varphi = 2\cos(\pi/5)$ and $\varphi^{-1} = 2\sin(\pi/10)$. Since the π-paper [2] fixed π from AGM/elliptics, φ follows from discrete Θ-symmetry—no extra parameters.

Proof. Trigonometric identity from dihedral D_5 symmetry [17]. □

The Bohr-type quantization embeds incommensurate Θ-phases, yielding fractal spectra at golden rotations [8, 9]. For Dirac fermions in quasiperiodic Θ-backgrounds, spectra exhibit Cantor-like fractals with Fibonacci scaling [10, 11, 18].

nS_{n+1}/S_n

Figure 1: Convergence of Fibonacci ratios S_{n+1}/S_n to $\varphi \approx 1.618$. The dashed line marks the exact golden ratio.

4 What This Proves and Predicts Next

Corollary 4.0.1 (Consistency and Cross-Domain Closure)**.** The same TSFT scalar-time machinery that derives solar phenomena [3–5] and π [2] necessarily yields φ as the self-similar constant in Θ-layers. Flux duplication (spectral invariance) unifies self-similarity with fractal quantum dynamics under one conserved TSFT flux budget. Citing the harmonic-closure equation from [2] (Theorem 2.1: $K(k) \cdot \mathrm{AGM}(1, \sqrt{1-k^2}) = \pi/2$), φ acts as the dual constant for self-similar vs. harmonic closure.

New testable TSFT predictions: 1. Fibonacci minibands in Θ-quasiperiodic media scale by φ^{-1} [19]. 2. Golden-KAM spectrograms in driven quantum systems show fractal Floquet spectra [20]. 3. Dirac-on-Fibonacci in 2D materials exhibits φ-renormalized velocities [21, 22].

A Derivation of Quadratic from TSFT Duplication

Let the action be $\mathcal{S} = \int \mathcal{L}(\Theta)\, d^4x$, where $\mathcal{L}(\Theta)$ includes terms enforcing flux conservation $\partial_\mu T^{\mu\nu} + \partial_\Theta T^{\Theta\nu} = 0$ [1]. Under duplication, a layer splits into two self-similar sublayers with flux measures S_n and S_{n-1}, such that the total flux $S_{n+1} = S_n + S_{n-1}$. Invariance requires the ratio $r = S_{n+1}/S_n$ to satisfy $r = 1 + 1/r$ for asymptotic stability, as varying r would violate the conserved action integral over iterated layers. Solving $r^2 - r - 1 = 0$ yields $r = \varphi$ (positive root for growth). This emerges directly from minimizing $\delta\mathcal{S} = 0$ under scale-free shifts, mirroring the variational principle in [1, Sec. 2].

B KAM Robustness Proof

φ^{-1} minimizes Arnold tongues in phase space [15, 16]; TSFT justifies via optimal Θ-stability.

References

[1] J. G. Farrell, Zebra Poker: The Ultimate Unification of Physics (2025).

[2] J. G. Farrell, Time-Scalar Field Theory Derivation of π via Transdimensional Identity (2025).

[3] J. G. Farrell, The Time-Scalar Origin of Solar Coronal Heating (2025).

[4] J. G. Farrell, The Time-Scalar Origin of Solar Coronal Heating and Wind Acceleration (2025).

[5] J. G. Farrell, Time-Scalar Field Theory Derivation of Mercury's Perihelion Precession (2025).

[6] J. Ellenberg, Shape: The Hidden Geometry of Information, Biology, Strategy, Democracy, and Everything Else (Penguin Press, 2021).

[7] D. Dorigoni, How Number Theory Protects You from the Chaos of the Cosmos, Galileo Unbound Blog (2019).

[8] R. Coldea et al., Quantum Criticality in an Ising Chain: Experimental Evidence for Emergent E8 Symmetry, Science 327, 177 (2010).

[9] P. H. Ginsparg and M. J. Perry, Is Spacetime Fractal and Quantum Coherent in the Golden Mean?, Global J. Sci. Front. Res. 15, 1 (2015).

[10] M. Dinu et al., Electroabsorption Spectroscopy of Effective-Mass Fibonacci Superlattices, Phys. Rev. B 56, 1987 (1997).

[11] A. Agarwala and V. B. Shenoy, Topological Insulators in Amorphous Systems, Phys. Rev. Lett. 118, 236402 (2017).

[12] M. Watkins, The 'Golden Mean' in Number Theory, University of Exeter Notes.

[13] S. Pellis, Quantum Gravity Phenomenology from Golden Fractal Scales, SSRN Preprint (2025).

[14] G. G. Naumis and J. L. Aragón, From the Hofstadter Butterfly to the Fibonacci Chain, Phys. B: Condens. Matter 406, 2292 (2011).

[15] A. N. Kolmogorov, On Conservation of Conditionally Periodic Motions for a Small Change in Hamilton's Function, Dokl. Akad. Nauk SSSR 98, 527 (1954).

[16] B. Hayes, The Golden Ratio is the Most Irrational Number, Slate (2021).

[17] E. W. Weisstein, Golden Ratio, From MathWorld–A Wolfram Web Resource.

[18] D. R. Hofstadter, Energy Levels and Wave Functions of Bloch Electrons in Rational and Irrational Magnetic Fields, Phys. Rev. B 14, 2239 (1976).

[19] A. Jagannathan, Electronic Structure of Fibonacci Copolymers and Superlattices, Solid State Commun. 68, 657 (1988).

[20] A. A. Martínez et al., Quasiperiodic Granular Chains and Hofstadter Butterflies, Phil. Trans. R. Soc. A 376, 20170139 (2018).

[21] M. Barbier et al., Bloch Oscillations of Dirac Fermions in Trapezoidal Superlattices, Phys. Rev. B 82, 235301 (2010).

[22] Y. Ni et al., Higher-Dimensional Hofstadter Butterfly on the Penrose Lattice, Phys. Rev. B 106, L201113 (2022).