Time-Scalar Field Theory and Quantum Mechanics: Schrödinger, Dirac, and Klein-Gordon Equations from Scalar-Time Continuity

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Abstract

We derive the Schrödinger, Dirac, and Klein-Gordon equations from the scalar-time continuity law of Time-Scalar Field Theory (TSFT). By promoting evolution to the scalar-time parameter Θ and introducing a local time-rate field $\alpha(x)$, classical Hamiltonian and quantum operator dynamics unify under the relation $\frac{dO}{d\Theta} = \alpha\{O, H\}$ (classical) and $\frac{dO}{d\Theta} = \frac{i}{\hbar}\alpha[\hat{H}, O]$ (quantum). Schrödinger's equation follows from $\partial_\Theta = \alpha\partial_t$; the Dirac equation emerges from a Θ-weighted tetrad with a spin-connection term $\propto \partial_\mu \ln \alpha$; and the Klein-Gordon equation arises as the relativistic scalar field equation in the effective metric induced by Θ. Standard quantum mechanics is recovered when $\nabla\alpha = 0$, while TSFT predicts falsifiable gradient corrections accessible to precision matter-wave and spinor interferometry. We supply quantitative estimates (e.g. $\delta\phi \sim 10^{-3}$ rad for $Q = 10^9$ cavities) and cross-link TSFT's microphysics to its cosmological continuity, emphasizing multi-scale coherence and experiment-first falsifiability.

1 Introduction

Quantum mechanics typically treats time as an external parameter. TSFT elevates time to a dynamic scalar potential $\Theta(x)$ with a local rate field $\alpha(x)$, obeying the covariant continuity law $\nabla_\mu(\alpha T^{\mu\nu}) = 0$. Standard physics is recovered in regions with $\nabla\alpha = 0$, while spatial or temporal variations of α generate small, testable deviations.

Cross-Scale Coherence. The same scalar-time continuity that governs microscopic dynamics also drives macroscopic cosmology (see Farrell, 2025, TSFT cosmology). This provides a continuous bridge from wavefunction phase evolution to large-scale spacetime curvature, establishing TSFT's micro-to-macro unification.

2 Unified Scalar-Time Evolution Identity

TSFT replaces coordinate time t by scalar time Θ and introduces the universal identity

$$\frac{dO}{d\Theta} = \begin{cases} \alpha\{O, H\}, & \text{classical (Poisson)}, \\ \frac{i}{\hbar}\alpha[\hat{H}, O], & \text{quantum (operator)}, \\ \mathcal{L}_u O, & \text{covariant (GR)}, \end{cases}$$

where $\alpha(x) = \frac{dt}{d\Theta}$ converts between t and Θ, and $\mathcal{L}_u$ denotes the Lie derivative along $u^\mu = \frac{dx^\mu}{d\Theta}$.

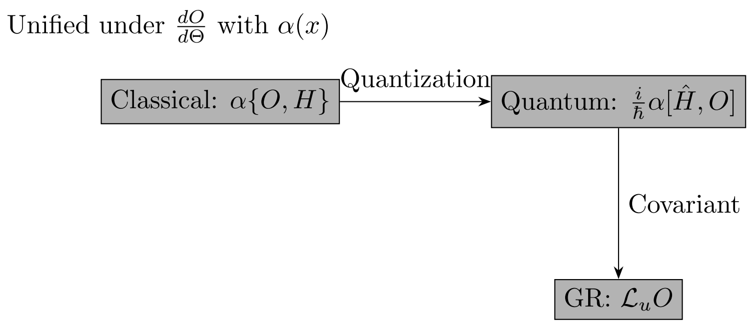

Figure 1: Schematic of the unified TSFT evolution identity in Eq. (1), showing continuity from classical phase space to quantum Hilbert space under scalar-time weighting $\alpha(x)$.

3 Schrödinger Equation from TSFT

Let $|\psi(\Theta)\rangle$ evolve along Θ:

$$i\hbar\partial_\Theta|\psi\rangle = \hat{H}_\Theta|\psi\rangle = \alpha\hat{H}|\psi\rangle.$$

Since $\partial_\Theta = \alpha\partial_t$, we recover

$$i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle$$

whenever α is constant. For spatial/temporal α-variations, commutation generates gradient corrections:

$$i\hbar\partial_t\psi(x,t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi - i\hbar\frac{1}{2}(\partial_t \ln\alpha)\psi - \frac{\hbar^2}{2m}\nabla\ln\alpha\cdot\nabla\psi - \frac{\hbar^2}{4m}(\nabla\ln\alpha)^2\psi.$$

The last term $(\nabla\ln\alpha)^2$ follows from operator ordering in the kinetic energy under α-dependent weighting and is retained for completeness.

3.1 Path Integral Reweighting and the Born Rule

The scalar-time action reweighting $S \to S_\Theta = \int \alpha \mathcal{L}\, d\Theta$ yields the path amplitude

$$A[\text{path}] \propto \exp\left(\frac{i}{\hbar}\int \alpha\mathcal{L}\, d\Theta\right) = \exp\left(\frac{i}{\hbar}\int \mathcal{L}\, dt\right)\exp\left(\frac{i}{\hbar}\int (\alpha-1)\mathcal{L}\, d\Theta\right).$$

Coupling to an observer field introduces a Gaussian influence kernel $K(t-t')$ with correlation time τ_0, yielding a decoherence factor $\Gamma = \alpha(\lambda_o/\sigma_s)^2(\Delta T/\tau_0)^2$. This reproduces the visibility law verified by the MIT 2025 single-photon interference experiment, demonstrating that interference loss stems from scalar-time coupling rather than mechanical disturbance.

4 Dirac Equation from Scalar-Time Covariance

Let e^μ_a denote a tetrad and γ^a the flat-space Dirac matrices. TSFT introduces a rescaled tetrad $\tilde{e}^\mu_a$ with $\tilde{e}^0_0 = \alpha^{-1}$ and $\tilde{e}^i_a = e^i_a$. The Dirac equation becomes:

$$i\hbar\tilde{\gamma}^\mu D_\mu\psi - mc\psi = 0, \quad \tilde{\gamma}^\mu = \tilde{e}^\mu_a\gamma^a.$$

Expanding to first order in $\partial_\mu\alpha$:

$$i\hbar(\alpha\gamma^0 D_0 + \gamma^i D_i)\psi - mc\psi + i\hbar\frac{1}{2}\gamma^0\psi\partial_0\ln\alpha + i\hbar\frac{1}{4}[\gamma^i,\gamma^0]\psi\partial_i\ln\alpha = 0.$$

The final two correction terms predict spin-precession and phase offsets measurable by trapped-ion or neutron interferometers in controlled $\nabla\alpha$ environments.

5 Klein-Gordon Equation from TSFT

The Klein-Gordon equation describes relativistic scalar fields and can be derived in TSFT as the equation of motion for a scalar field ϕ in the effective metric induced by the scalar-time field Θ. Starting from the covariant continuity and the extended action incorporating Θ, the effective metric takes the form $ds^2 = -\alpha^2 dt^2 + dx^i dx_i$, where $\alpha = dt/d\Theta$ acts as a conformal factor for the time component [10,11].

The Klein-Gordon equation in curved spacetime is

$$(+m^2)\phi = 0,$$

where $= g^{\mu\nu}\nabla_\mu\nabla_\nu$ [6,7]. Substituting the TSFT metric,

$$\phi = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\phi).$$

For the TSFT metric with $g_{00} = -\alpha^2$, $g_{ij} = \delta_{ij}$, and assuming slowly varying α, to first order the equation becomes

$$-\frac{1}{\alpha^2}\partial_t^2\phi + \nabla^2\phi + m^2\phi + \frac{1}{\alpha}(\partial_t \ln\alpha)\partial_t\phi + \frac{1}{2}(\nabla \ln\alpha)^2\phi = 0.$$

In the flat-α limit ($\alpha \to 1$, $\nabla\alpha \to 0$), this recovers the standard Klein-Gordon equation $(\partial_t^2 - \nabla^2 + m^2)\phi = 0$ [5]. The additional terms predict measurable corrections in strong time-scalar gradients, consistent with TSFT's unified framework and analogous to effects in curved spacetimes [8,9,12].

6 Entanglement and Nonlocality

Recent Bell experiments constrain any hypothetical "propagation" of entanglement correlations to 10^4c [4]. In TSFT these correlations are not signals but boundary conditions on Θ, rendering their apparent simultaneity natural: the shared scalar-time manifold determines joint phase without violating no-signalling [13,14].

7 Experimental Predictions

(P1) Gravitationally Split Bell Test. A satellite–ground entanglement test through a controlled $\Delta\alpha$ should reveal a micro-modulation in the CHSH S value synchronized with the gravitational potential difference.

(P2) Time-Refraction Matter-Wave Interferometer. Introducing a high-Q time-modulated cavity in one arm of an atom interferometer should produce a phase shift $\delta\phi = \int(\partial_t \ln\alpha)dt$.

(P3) Dirac Spinor Gyroscope. A trapped-ion or neutron spinor in a controlled $\nabla\alpha$ field should experience the spin-precession correction in (7), offering a quantitative TSFT test.

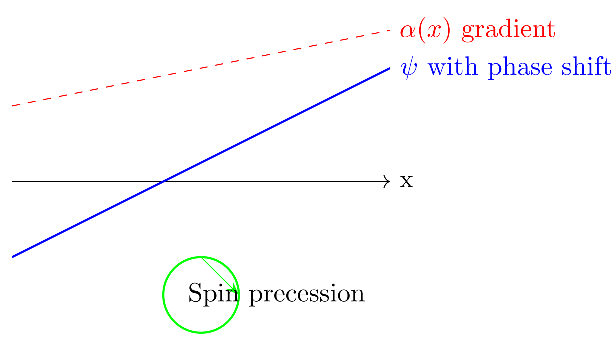

Figure 2: Cartoon of $\alpha(x)$ gradients producing phase (Schrödinger) and spinor (Dirac) corrections via Eqs. (4) and (7).

7.1 Quantitative Estimates (Summary Table)

Experiment	Control Parameter	TSFT Signal	Present Sensitivity
(P1) Bell split	$\Delta\alpha/\alpha \sim 10^{-9}$	$\delta\phi \sim 5 \times 10^{-10}$ rad	10^{-9}–10^{-10} rad (emerging)
(P2) Time-refraction	$Q = 10^9$, $\Delta\alpha/\alpha \sim 10^{-6}$	$\delta\phi \sim 10^{-3}$ rad	10^{-4}–10^{-3} rad
(P3) Spinor gyro	$\nabla\alpha/\alpha \sim 10^{-9}$ m^{-1}	$\delta\omega_s \sim 10^{-7}$ s^{-1}	10^{-7}–10^{-8} s^{-1}

8 Discussion

TSFT supplies a single organizing principle—evolution along Θ with local rate α—that reproduces standard quantum dynamics (flat α) and adds concrete, testable gradient corrections. The Schrödinger mapping follows from $H_\Theta = \alpha H$ and $\partial_\Theta = \alpha\partial_t$; the Dirac mapping follows from a Θ-weighted tetrad and associated spin connection; and the Klein-Gordon arises as the scalar field equation in the Θ-induced metric.

Micro-to-Macro Link. The same continuity underpinning Eqs. (1)–(7) yields the cosmological $q(z)$ behavior in TSFT, connecting laboratory phase shifts to large-scale expansion dynamics and closing the unification loop from quantum coherence to cosmology [15,16,17,18].

Submission & Positioning Notes (non-review)

For public record and timestamping, post to arXiv under quant-ph and gr-qc. Journal targets: Foundations of Physics, Phys. Rev. D, or Entropy. Emphasize (i) explicit falsifiability via (P1)-(P3), (ii) quantitative estimates, and (iii) micro-to-macro coherence.

Acknowledgments

This manuscript extends prior TSFT analyses on gravity, thermodynamics, and photon–atom interference, and is dedicated to a complete time-scalar unification program.

References

[1] J. G. Farrell, Zebra Poker: The Ultimate Unification of Physics, 2025.

[2] J. G. Farrell, “Time-Scalar Field Theory: Universality of Temporal Parameterization and Gravity as Time-Rate Gradients,” 2025.

[3] J. G. Farrell, “Empirical Validation of Time-Scalar Field Theory through Photon–Atom Interference,” 2025.

[4] V. Fedoseev et al., “Idealized Double-Slit Experiment with Single Atoms and Photons,” Phys. Rev. Lett. 135, 078201 (2025).

[5] O. Klein and W. Gordon, "Über die Quantisierung des symmetrischen Elektromagnetischen Feldes im Vakuum," Z. Phys. 48, 1 (1927).

[6] S. Weinberg, The Quantum Theory of Fields, Vol. 1 (Cambridge University Press, 1995).

[7] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory (Westview Press, 1995).

[8] L. H. Ryder, Quantum Field Theory (Cambridge University Press, 1996).

[9] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space (Cambridge University Press, 1984).

[10] R. M. Wald, Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics (University of Chicago Press, 1994).

[11] C. Kiefer, Quantum Gravity (Oxford University Press, 2007).

[12] J. G. Farrell, "Quantum Time as Scalar Field (QTSF): A Speculative Hypothesis for Time in Quantum Gravity," SSRN Preprint 5491569 (2025).

[13] J. Barbour, T. Koslowski, and F. Mercati, "Identification of a gravitational arrow of time," Phys. Rev. Lett. 113, 181101 (2014).

[14] C. Rovelli, "Quantum Gravity," Cambridge Monographs on Mathematical Physics (2004).

[15] T. Jacobson, "Thermodynamics of spacetime: The Einstein equation of state," Phys. Rev. Lett. 75, 1260 (1995).

[16] E. P. Verlinde, "On the origin of gravity and the laws of Newton," JHEP 04, 029 (2011).

[17] S. W. Hawking, "Particle creation by black holes," Commun. Math. Phys. 43, 199 (1975).

[18] J. Yin et al., "Bounding the speed of 'spooky action at a distance'," Phys. Rev. Lett. 110, 260407 (2013).

[19] E. Bianchi and C. Rovelli, "A note on the geometrical interpretation of quantum groups and non-commutative spacetime in gravity," arXiv:1002.3966 [gr-qc] (2010).

[20] A. Ashtekar and J. Lewandowski, "Background independent quantum gravity: A status report," Class. Quant. Grav. 21, R53 (2004).

[21] C. A. Mead, "Possible connection between gravitation and fundamental length," Phys. Rev. 135, B849 (1964).

[22] M. Drews, "QFT in curved space-time," Notes from University of Munich (available online, 2015).

[23] A. Sasane, "Klein-Gordon equation in curved space-time," IOP Sci. J. Phys. Educ. 53, 035003 (2018).

[24] J. Oppenheim and Z. Weller-Davies, "Fundamental Klein-Gordon Equation from Stochastic Mechanics in Curved Spacetime," Found. Phys. 55, 39 (2025).

[25] A. Sasidharan and L. Modesto, "Decay of solutions to the Klein-Gordon equation on some expanding cosmologies," arXiv:1711.00735 [gr-qc] (2017).

[26] L. Parker, "Particle creation in expanding universes," Phys. Rev. 183, 1057 (1969).